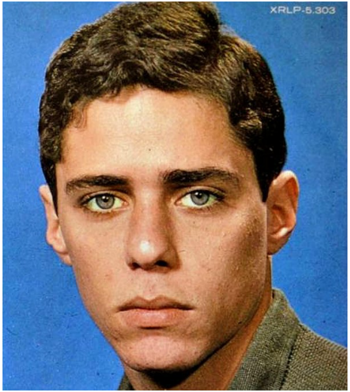
Quantum cryptography from weaker computational assumptions

Alex Bredariol Grilo

Quantum information vs. cryptography

Quantum information vs. cryptography



Quantum helps malicious parties

Quantum information vs. cryptography

Quantum helps honest parties   Quantum helps malicious parties

Quantum information vs. cryptography

Quantum helps honest parties Quantum helps malicious parties

How do quantum resources allow us to achieve better cryptographic protocols?

Quantum 101 (simplified)

Quantum mechanics

Quantum states	Evolution	Measurements

Quantum 101 (simplified)

Quantum mechanics

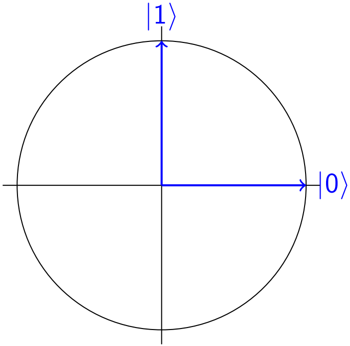
Quantum states	Evolution	Measurements

Quantum 101 (simplified)

Quantum mechanics

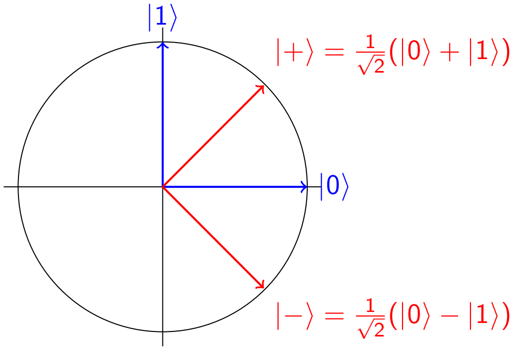
Quantum states | Evolution | Measurements

States



Quantum 101 (simplified)

States



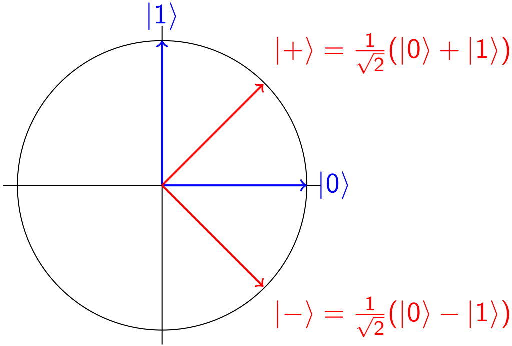
Measurements

Quantum 101 (simplified)

Quantum mechanics

Quantum states   Evolution   Measurements

States



Measurements

- Computational basis $\rightarrow$

Quantum 101 (simplified)

Quantum mechanics

Quantum states   Evolution   Measurements

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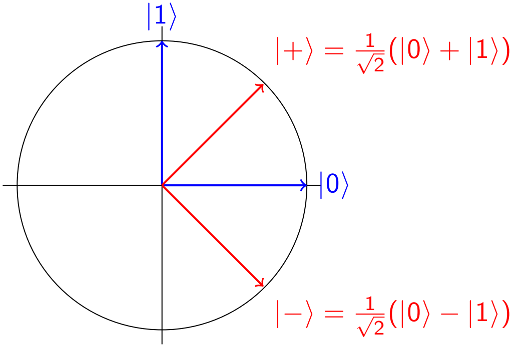


Measurements

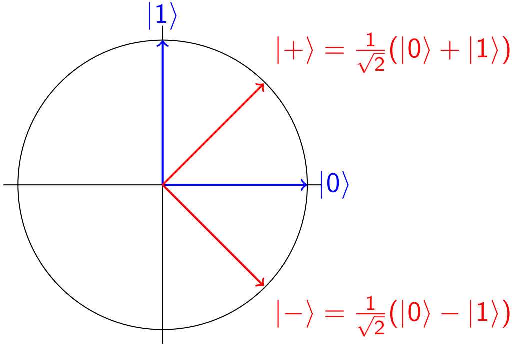
- Computational basis $\rightarrow$
  - Meas. $|0\rangle$, the outcome is 0 w.p. 1
  - Meas. $|1\rangle$, the outcome is 1 w.p. 1

Quantum 101 (simplified)

Quantum mechanics

Quantum states | Evolution | Measurements

States



Measurements

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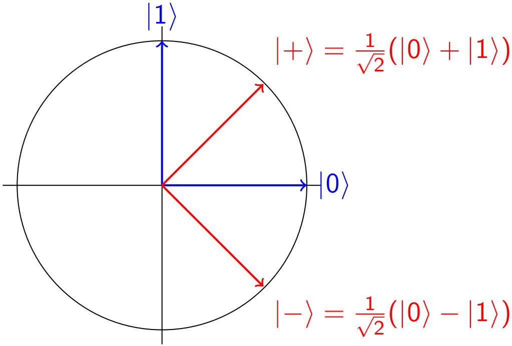
Quantum 101 (simplified)

States



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Quantum 101 (simplified)

Quantum mechanics

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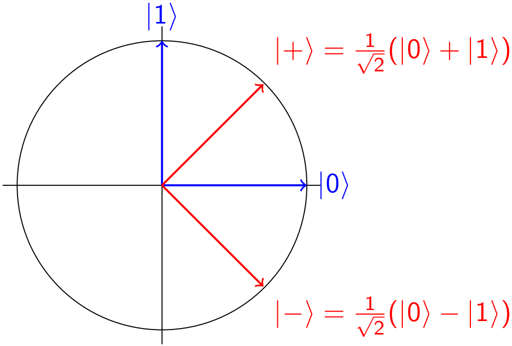

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Quantum 101 (simplified)

Quantum mechanics

Quantum states | Evolution | Measurements

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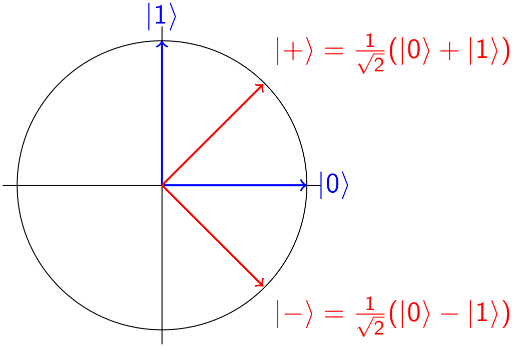

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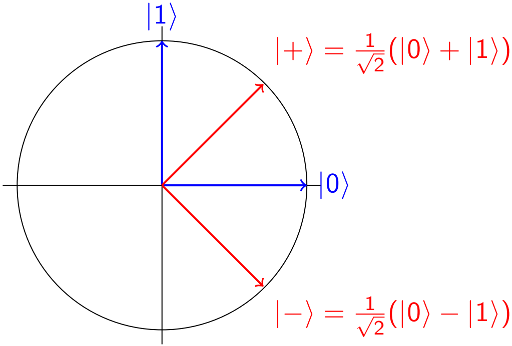
Quantum 101 (simplified)

States



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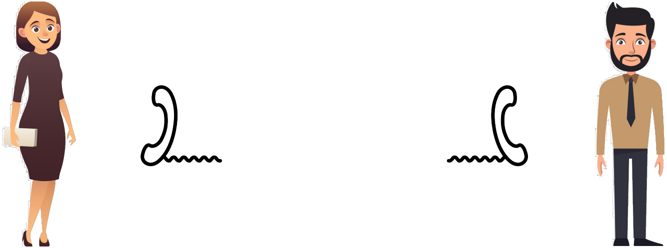
State collapses after measurements

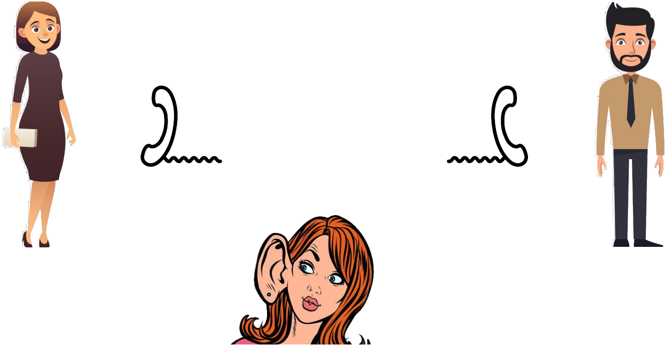
Key agreement

Key agreement



Goal: Alice and Bob want to share a common random key k by the phone

Key agreement



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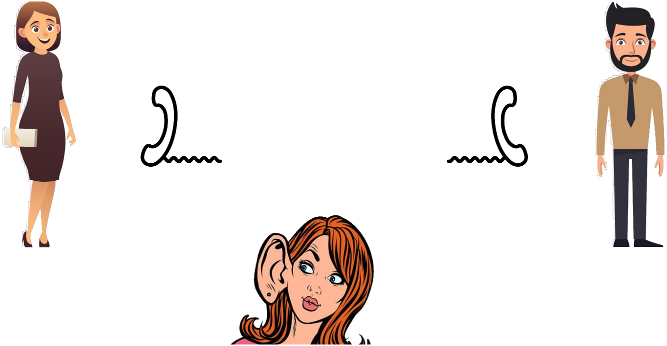

Goal: Alice and Bob want to share a common random key k by the phone
Security: They want k to be unknown to potential eavesdroppers
Classical information-theoretically secure key agreement is impossible!

Quantum-key distribution (simplified)

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	Basis
$\lvert\phi_1\rangle = \lvert+\rangle$	1
$\lvert\phi_2\rangle = \lvert 0\rangle$	0
$\lvert\phi_3\rangle = \lvert 1\rangle$	0
$\lvert\phi_4\rangle = \lvert 0\rangle$	0
$\lvert\phi_5\rangle = \lvert-\rangle$	1
$\lvert\phi_6\rangle = \lvert-\rangle$	1

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$$\xrightarrow{|{+}010{-}{-}\rangle}$$

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$$\xrightarrow{\lvert+010--\rangle}$$

Basis	Outcome
1	$+$
1	$-$
0	1
0	0
0	0
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$$\xrightarrow{\quad |+010--\rangle \quad}$$

$$\xleftarrow{\quad 110001 \quad}$$

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$\xrightarrow{\lvert +010--\rangle}$

$\xleftarrow{110001}$

$\xrightarrow{100011,\ \mathrm{T} = \{1,4\}}$

Basis	Outcome
1	$+$
1	$-$
0	1
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0	0
1	$-$

Quantum-key distribution (simplified)

	Basis
$\color{blue}{\lvert\phi_1\rangle = \lvert+\rangle}$	$\color{blue}{1}$
$\lvert\phi_2\rangle = \lvert 0\rangle$	0
$\color{blue}{\lvert\phi_3\rangle = \lvert 1\rangle}$	$\color{blue}{0}$
$\color{blue}{\lvert\phi_4\rangle = \lvert 0\rangle}$	$\color{blue}{0}$
$\lvert\phi_5\rangle = \lvert -\rangle$	1
$\color{blue}{\lvert\phi_6\rangle = \lvert -\rangle}$	$\color{blue}{1}$

$$\xrightarrow{\quad\lvert+010--\rangle\quad}$$

$$\xleftarrow{\quad 110001\quad}$$

$$\xrightarrow{\quad 100011,\ \mathrm{T} = \{1,4\}\quad}$$

$$\xleftarrow{\quad +0\quad}$$

Basis	Outcome
$\color{blue}{1}$	$\color{blue}{+}$
1	$-$
$\color{blue}{0}$	$\color{blue}{1}$
$\color{blue}{0}$	$\color{blue}{0}$
0	0
$\color{blue}{1}$	$\color{blue}{-}$

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$$\xrightarrow{\;|{+}010--\rangle\;}$$

$$\xleftarrow{\;110001\;}$$

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Basis	Outcome
1	+
1	−
0	1
0	0
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1	−

- Intuitively, if Eve tries to eavesdrop the quantum state, it collapses

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  - Complete protocol and formal security proof is more cumbersome

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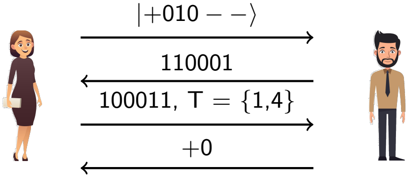

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- Can we achieve other protocols such as bit-commitment, MPC,... unconditionally?

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$\xrightarrow{\lvert +010--\rangle}$

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$\xrightarrow{100011,\ T=\{1,4\}}$

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- No! [M'97, LC'97]

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  - Complete protocol and formal security proof is more cumbersome
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- No! [M'97, LC'97]

What if we use computational assumptions?

Classical cryptographic primitive/assumptions

Public-key encryption

Functional encryption

Oblivious transfer

indistinguishable Obfuscation

Secret-key encryption

Two-party computation

Witness encryption

Multi-party computation

One-way functions

Pseudo-random number generators

Zero-knowledge proof systems

How to propose implementations and prove their security?

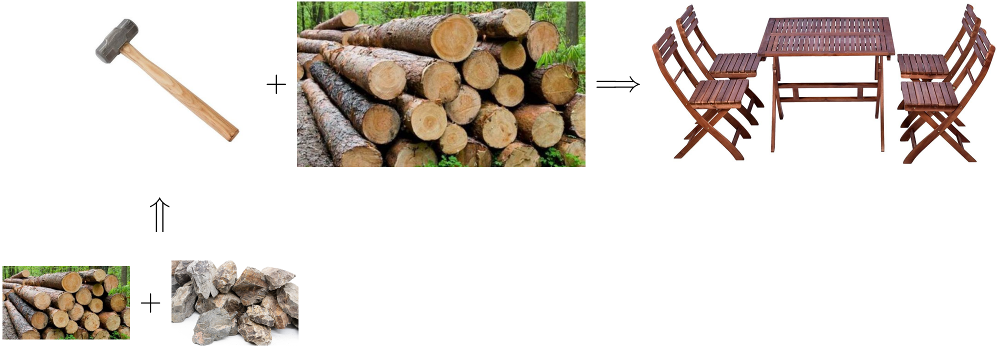
Reductions

Reductions

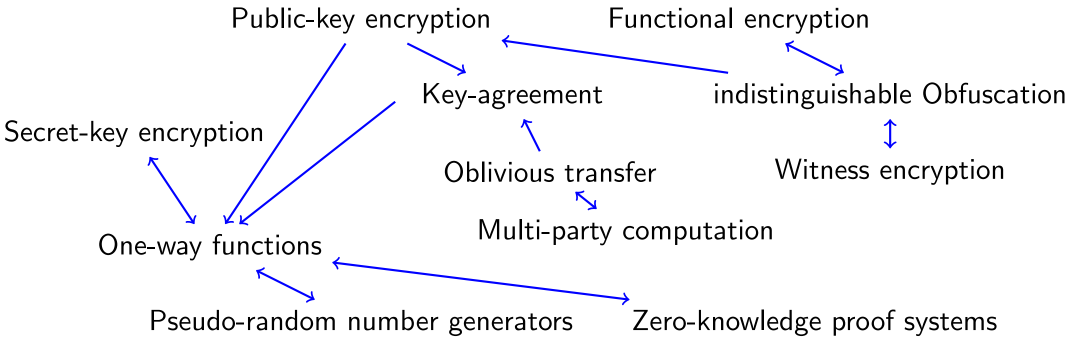
$+$

$\Longrightarrow$

Reductions



Primitives



Public-key encryption
Functional encryption
Key-agreement
indistinguishable Obfuscation
Secret-key encryption
Oblivious transfer
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Multi-party computation
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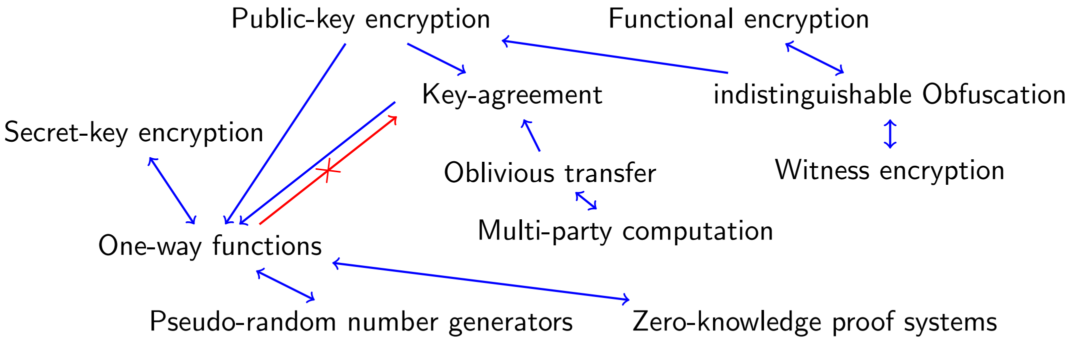
Pseudo-random number generators
Zero-knowledge proof systems

Primitives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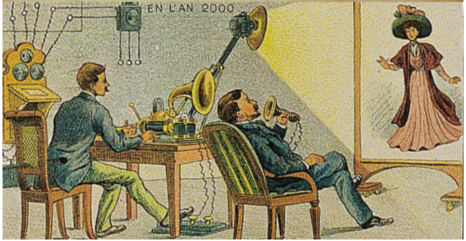
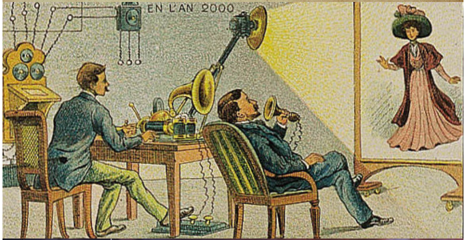
Minicrypt: OWFs exist



Cryptomania: PKE schemes exist

Minicrypt: OWFs exist



Cryptomania: PKE schemes exist

Obfutopia: iO exists

… if crypto is possible

Algorithmica(+Heuristica): We can solve NP (in practice)

Pessiland: We cannot solve NP and OWFs do not exist

Minicrypt

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One-way function f

For every polynomial-time adversary $\mathcal{A}$, and polynomial ℓ:

$$\Pr_x[\mathcal{A}(f(x)) \in f^{-1}(x)] \leq negl(n).$$

Minicrypt

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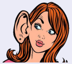
Symmetric-key encryption

sk

$c = Enc_{sk}(m)$

$\longleftarrow$



sk

$m' = Dec_{sk}(c)$

Minicrypt

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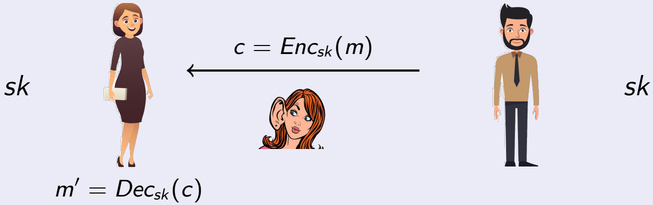

Pseudo-random function $\{f_k\}_k$

For every polynomial-time adversary $\mathcal{A}$:

$$|\Pr_k[\mathcal{A}^{f_k}() = 1] - \Pr_{f \sim U}[\mathcal{A}^f() = 1]| \le negl(n).$$

This talk

1. Quantum protocols for public-key encryption
2. Quantum protocols for multi-party computation
3. Weaker assumptions in the quantum world

Quantum protocols for public-key encryption

Public-key encryption

Public-key encryption

$$Gen(1^n) = (sk, pk)$$

sk

Public-key encryption

pk

sk

Public-key encryption

pk

sk

$c = Enc_{pk}(m)$

Public-key encryption

pk

sk

$c = Enc_{pk}(m)$

$m' = Dec_{sk}(c)$

Public-key encryption

pk

sk

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$\longleftarrow$

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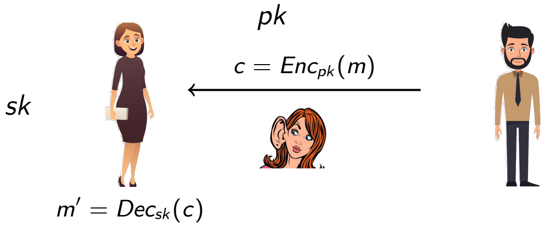
Correctness

$$Dec_{sk}(Enc_{pk}(m)) = m$$

Public-key encryption



Correctness

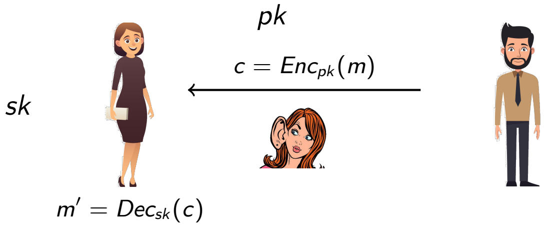
$$Dec_{sk}(Enc_{pk}(m)) = m$$

Security (simplified)

For every polynomial-time adversary $\mathcal{A}$:

$|\Pr[\mathcal{A}(pk, Enc_{pk}(0)) = 1] - \Pr[\mathcal{A}(pk, Enc_{pk}(1)) = 1]| \leq negl(n)$.

Public-key encryption



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Theorem [IR'89]

PKE cannot be built from OWF in a black-box way

Public-key encryption with quantum public keys

Public-key encryption with quantum public keys

$$Gen(1^n) = (sk, |pk\rangle^{\otimes \ell})$$

sk

Public-key encryption with quantum public keys

$|pk\rangle^{\otimes \ell}$

sk

Public-key encryption with quantum public keys

$|pk\rangle^{\otimes \ell}$

sk

$c = Enc_{|pk\rangle}(m)$

Public-key encryption with quantum public keys

$|pk\rangle^{\otimes \ell}$

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sk

$\longleftarrow$

$m' = Dec_{sk}(c)$

Public-key encryption with quantum public keys

$|pk\rangle^{\otimes \ell}$

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Correctness

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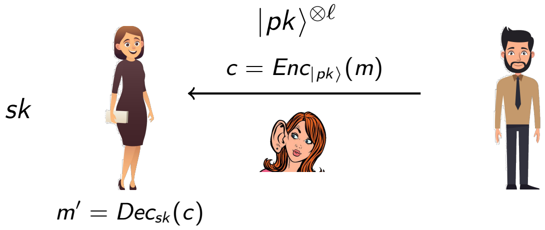
Public-key encryption with quantum public keys



Correctness

$$Dec_{sk}(Enc_{|pk\rangle}(m)) = m$$

Security (simplified)

For every polynomial-time adversary $\mathcal{A}$, and polynomial ℓ:
$|\Pr[\mathcal{A}(|pk\rangle^{\otimes \ell}, Enc_{|pk\rangle}(0)) = 1] - \Pr[\mathcal{A}(|pk\rangle^{\otimes \ell}, Enc_{|pk\rangle}(1)) = 1]| \leq negl(n)$.

QPKE from OWF [BGHMSVW'23]

QPKE from OWF [BGHMSVW'23]

Construction

QPKE from OWF [BGHMSVW'23]

Construction

- $sk = k$ and $|pk\rangle = \frac{1}{\sqrt{2^n}} \sum_x |x\rangle |PRF_k(x)\rangle$

QPKE from OWF [BGHMSVW'23]

Construction

- $sk = k$ and $|pk\rangle = \frac{1}{\sqrt{2^n}} \sum_x |x\rangle |PRF_k(x)\rangle$
- $Enc_{|pk\rangle}(m)$:
  1. Measure $|pk\rangle$ and get $(x^*, PRF_k(x^*))$
  2. $c = (x^*, c^* = SE.Enc_{PRF_k(x^*)}(m))$

QPKE from OWF [BGHMSVW'23]

Construction

- $sk = k$ and $|pk\rangle = \frac{1}{\sqrt{2^n}} \sum_x |x\rangle|PRF_k(x)\rangle$
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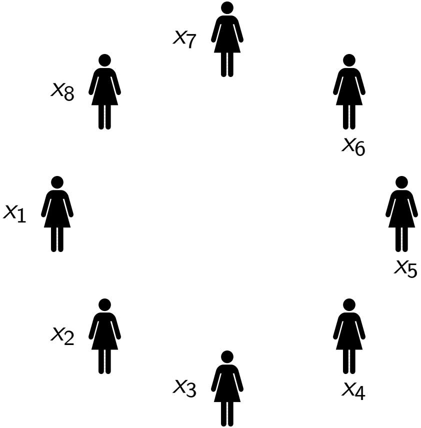
- Security comes from SKE, PRF and randomness of quantum measurements

Further results

- Impossibility of information-theoretically secure QPKE [B**G**HMSVW'23]
- QPKE from pseudo-random states (with special properties) [B**G**HMSVW'23]
- Quantum trapdoor functions and quantum PKE [C'23]
- Tamper-resilient QPKE from OWF [KMNY'23]
- Non-interactive KE from OWF [MW'23]

Quantum protocols for multi-party computation

Multi-party computation



Goal: Compute $f(x_1, ..., x_8)$ without revealing their input

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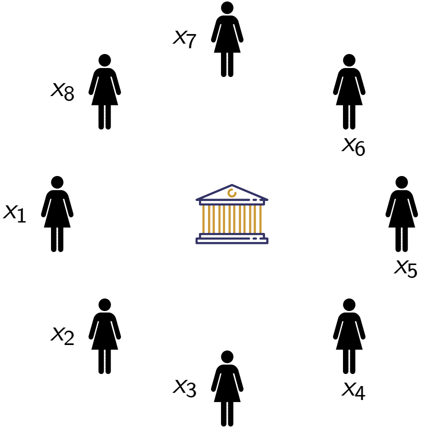

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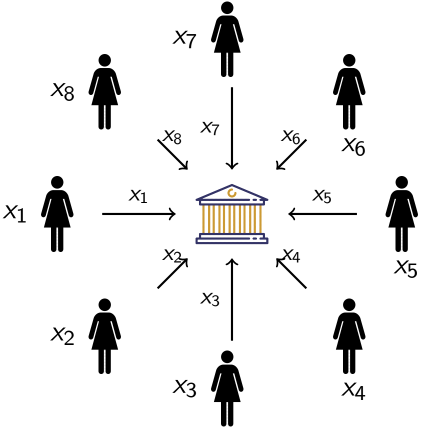
Ideal world

Multi-party computation



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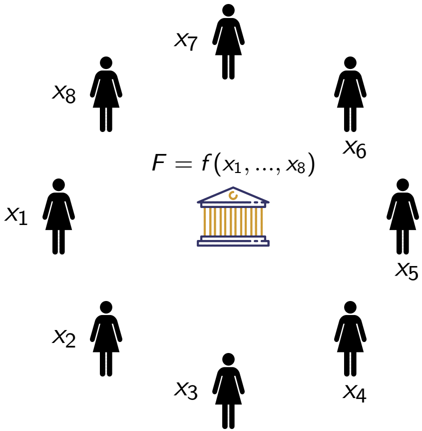

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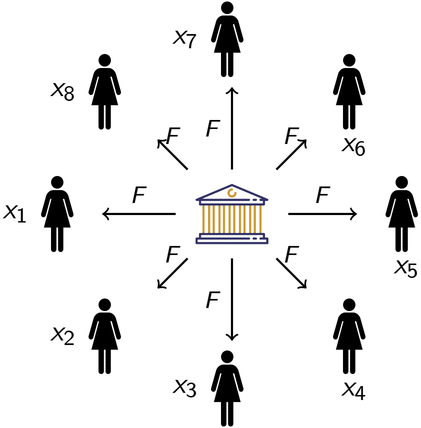

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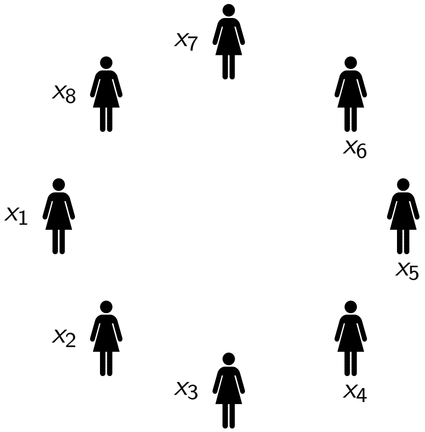

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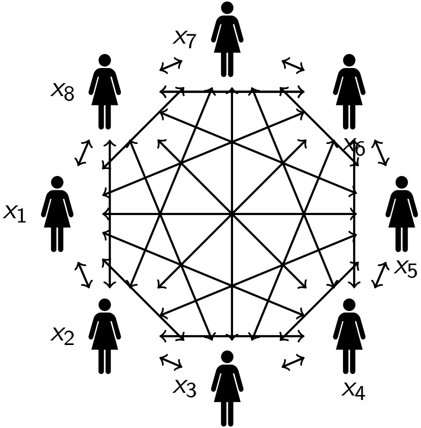

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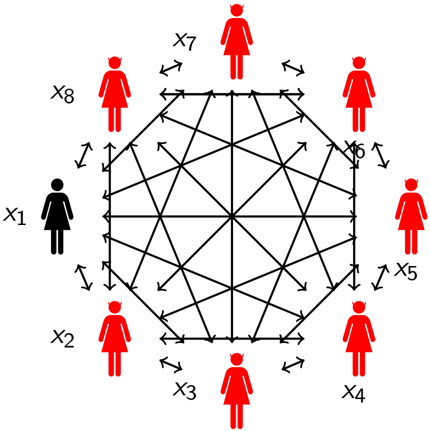

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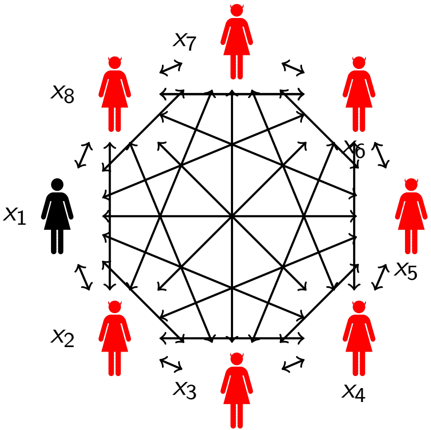

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Theorem [MMP'12]

MPC cannot be built from OWF in a black-box way

Oblivious transfer

Oblivious transfer

Ideal functionality

S $\qquad$ $\mathcal{F}_{\texttt{ot}}$ $\qquad$ R

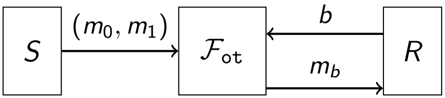
Oblivious transfer

Ideal functionality

$$\begin{tikzcd} \boxed{S} \arrow[r, "(m_0, m_1)"] & \boxed{\mathcal{F}_{\texttt{ot}}} & \boxed{R} \arrow[l, "b"'] \end{tikzcd}$$

Oblivious transfer

Ideal functionality



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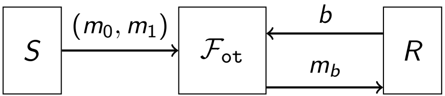
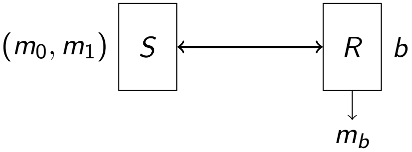


Real world



MPC from Quantum+OWF

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Corollary

Quantum protocol for MPC from OWF

CK/BBCS protocol (I)

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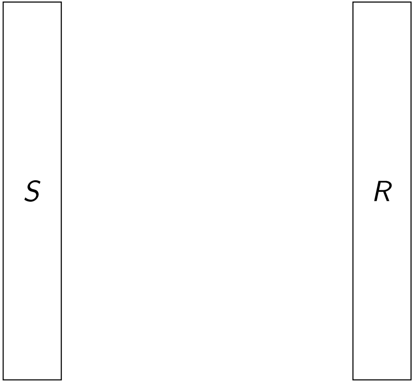
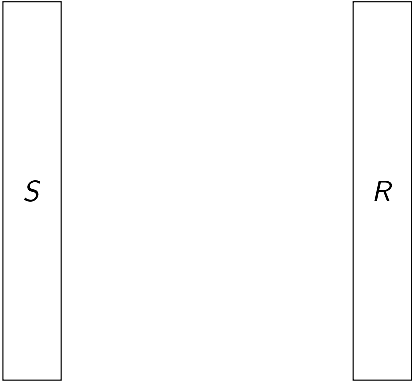


CK/BBCS protocol (I)

$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$



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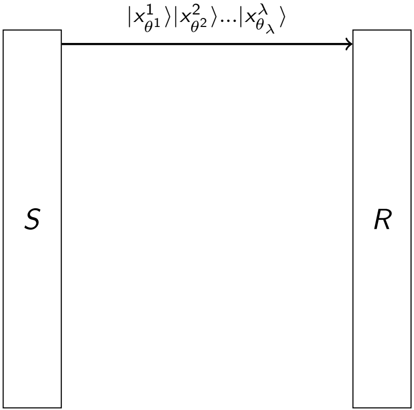

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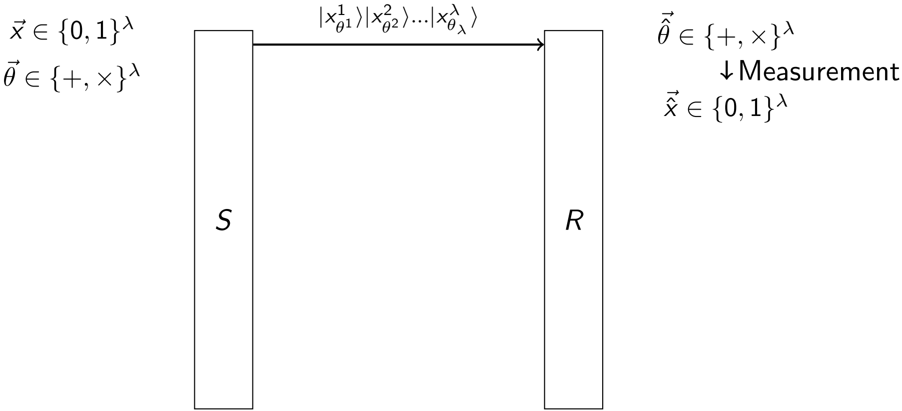

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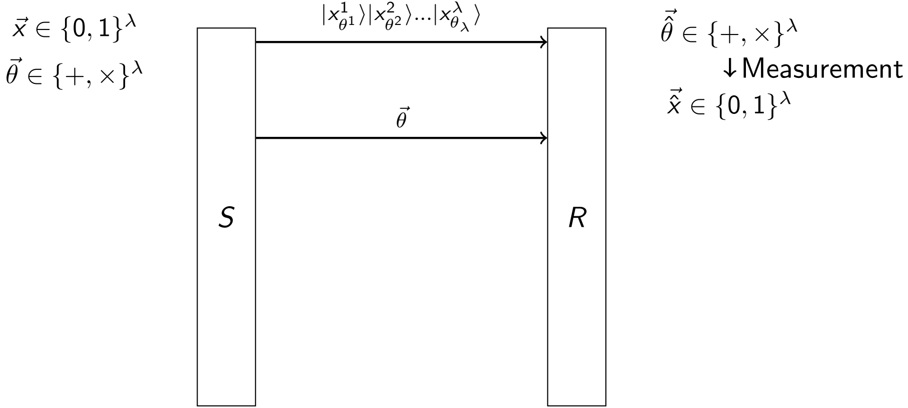

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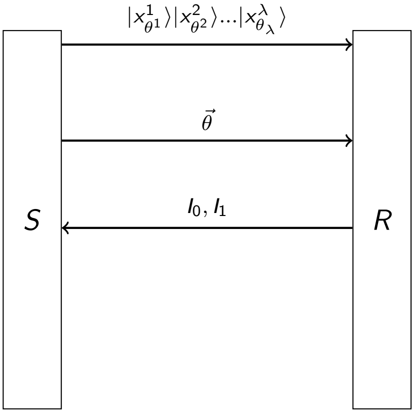

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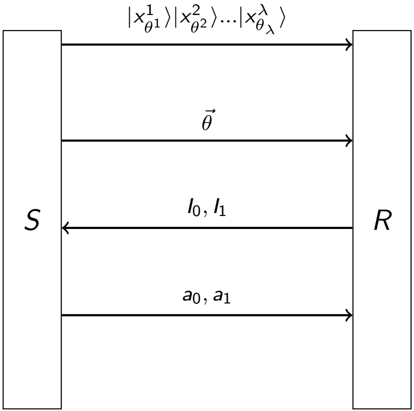

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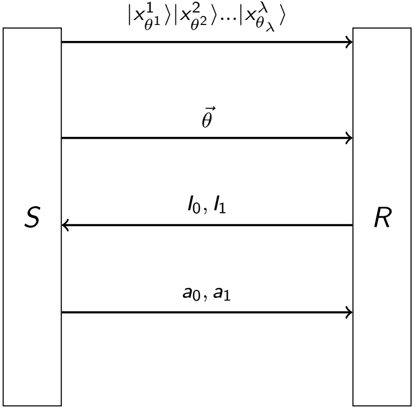

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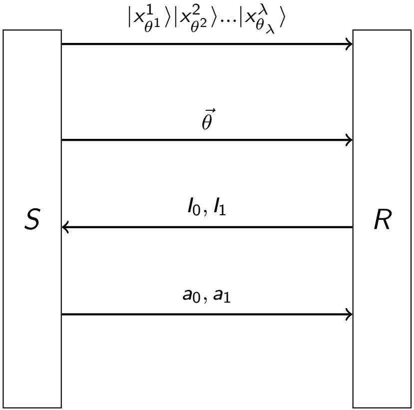

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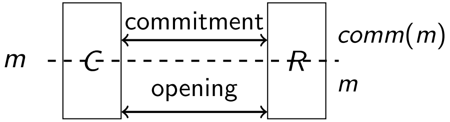
Attack for malicious receiver: $\tilde{R}$ waits $\vec{\theta}$ to measure the qubits using the right basis

Bit-commitment with simulation security



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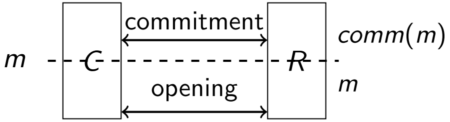

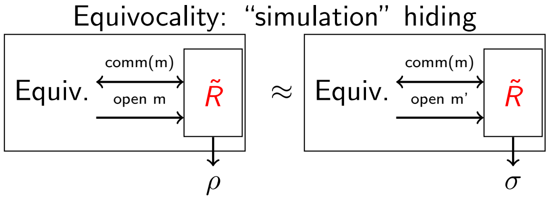

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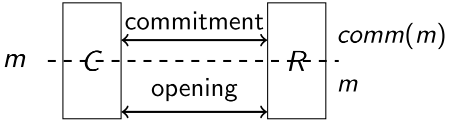
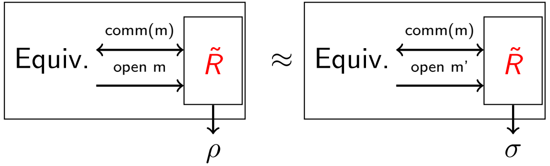
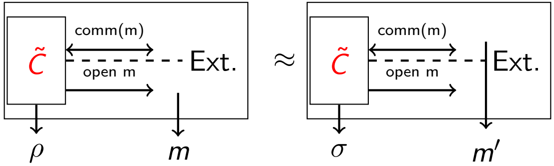


Equivocality: "simulation" hiding



Extractability: "simulation" binding



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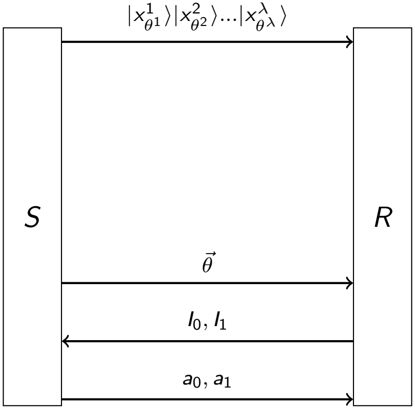

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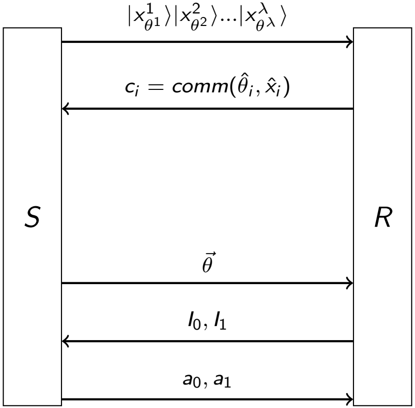

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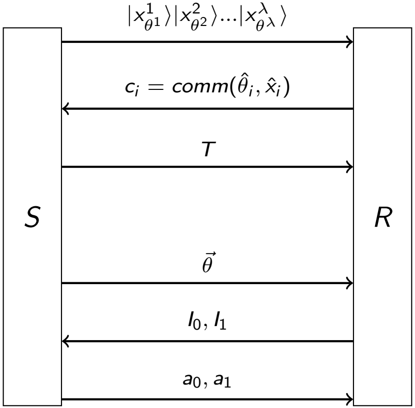

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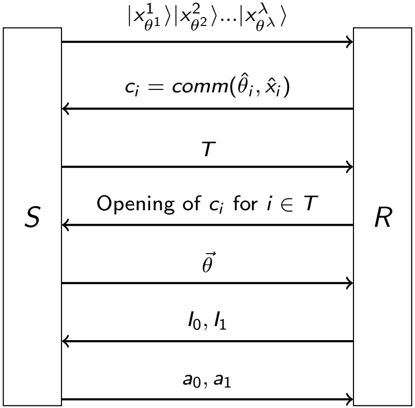

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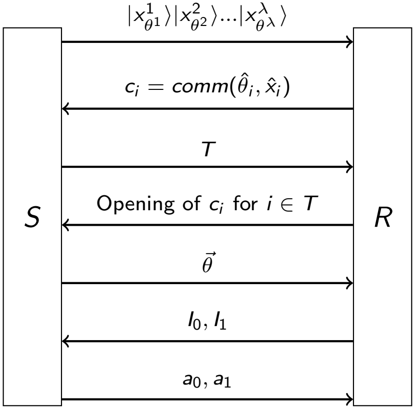

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Implemententing commitment scheme with simulation security from OWF

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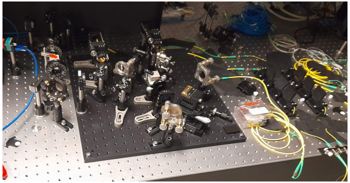
[BCKM21]	[GLSV21]
1. (Black-box) equivocality compiler	1. Equivocal commitment from Naor's commitment and zero-knowledge
2. Extractable commitment from equivocal commitment and quantum communication	2. Unbounded-simulator OT from equivocal commitment
	3. Extractable and equivocal commitment from unbounded-simulator OT and quantum communication

Features:

[BCKM21]	[GLSV21]
• **Black-Box** use of one-way functions	• **Constant-Round** OT in the CRS model
• **Statistical** security against malicious receiver	• **Statistically binding** extractable commitment

Further results

- QPKE from pseudo-random states (with special properties) [AQY'22]
- Practical protocols [D**G**ILYY'23 – on-going]
- Experimental implementation [IYYL**G**D'24 – on-going]



Weaker assumptions in the quantum world

Pseudo-random states

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Pseudo-random states $\{|\psi_k\rangle\}_k$

For every polynomial-time adversary $\mathcal{A}$, and polynomial ℓ:

$$|\Pr_k[\mathcal{A}(|\psi\rangle^{\otimes \ell}) = 1] - \Pr_{|\phi\rangle \sim \text{Haar}}[\mathcal{A}(|\phi\rangle^{\otimes \ell}) = 1]| \leq negl(n).$$

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- Variants of PRS can be built from OWF [AGQY'22],[BBSS'23]
- Constructions of strong primitives from PRS [AQY'22,...]

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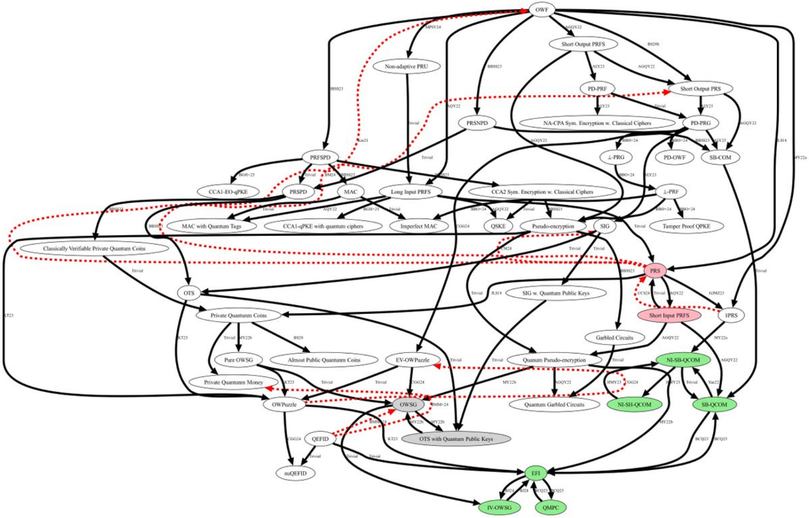
OWF might not be the weakest computational assumption with quantum resources

Microcrypt? Nanocrypt?



Conclusions and open questions

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Thank you for your attention!