Physical Unclonable Functions (PUFs): Signal Processing and Information-Theoretic Aspects

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- > Part I: Signal Processing for PUFs with Security Guarantees
- > Part II: Optimal Error-Correcting Code Designs for PUFs

> Signal Processing for PUFs with Security Guarantees

Digital Secrecy and Privacy Example

 Digital devices have to be protected/secured/authenticated/identified, similar to individuals, to provide security for Things and their Internet (in order to protect device owners and businesses):
 ⇒ Hardware "Fingerprint" via a PUF





Other Applications of PUFs

- 6G mobile devices that use SRAM outputs, available in mobile devices, as a PUF to extract secret keys;
- A PUF in a USB token to encrypt user data before uploading it to the cloud;
- System developers want to mutually authenticate an FPGA chip and the IP components in the chip, while IP developers want to protect the IP. One symmetric cipher and one PUF can achieve these.

A Brief Definition of PUF

- A PUF is a challenge-response-mapping embodied by a physical device (e.g., digital circuit outputs) such that it is
 - easy and fast for the physical device to evaluate the PUF response;
 - hard for an attacker to determine the PUF response to a randomly chosen challenge, even if the attacker has access to a set of challenge-response pairs.
- PUFs are significantly cheaper and safer alternatives to storing keys in a non-volatile memory and they are the only alternative that fits perfectly to the hardware requirements of IoT networks to provide security to all digital devices, as well as to the whole network.
- PUFs can be used to also secure 6G networks, Cyber devices, Digital twins, Metaverse, etc.

A Simple PUF with Continuous-Valued Outputs

Ring Oscillator (RO) PUFs

• A delay-based intrinsic PUF scheme uses the **random variations in the oscillation frequencies** of ROs to generate a secret key.



A Simple PUF with Continuous-valued Outputs II

RO PUFs

- Source of randomness: Uncontrollable silicon process variations on digital components' delays;
- > Hard macro designs are used for each RO: identical implementations;
- Temperature and voltage effects are orders of magnitude greater than the random variations in RO outputs;
- Correlations in RO outputs decrease entropy in the extracted bit sequence;
- > There is **noise** in every measurement of digital circuits.

Secret Key Generation with RO PUFs



- F: Real-valued Oscillation Frequencies
- B: Uniform Bit Sequence
- W: Side Information
- N: Noise
- E: Error Vector

Fuzzy Commitment Scheme



Secret key S and helper data W have to be independent,

- ▶ Block error probability should satisfy, e.g., $P_B = \Pr[S \neq \hat{S}] \le 10^{-9}$;
- S should be uniformly random with entropy, e.g., $H(S) \ge 256$ bits.

Main Components to Design



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How to Solve the 3 Problems Simultaneously



- ► Apply a **transform** $T_{r \times c}(\cdot)$ to decorrelate \widetilde{X}^{L} ;
- Histogram equalization converts each transform-coefficient T output into a standard (Gaussian) random variable;

How to Solve the 3 Problems Simultaneously II

> Apply scalar quantizers that satisfy the uniformity property:

$$\Pr[\text{Quant}(\widehat{T}_i) = (q_1, q_2, \dots, q_{K_i})] = \frac{1}{2^{K_i}} \quad \text{for} \quad i = 1, 2, \dots, L$$
 (1)



How to Solve the 3 Problems Simultaneously III



- The noise components have zero mean, so use Gray mapping to extract bit sequences from quantized outputs;
- ► Concatenate all extracted bits to obtain X^N/Y^N ;
- Firror symbols $E_i = X_i \oplus Y_i$ need not be independent or identically distributed.

> Model transform coefficients T as, e.g., **truncated Gaussian** distributions. Then, quantization boundaries, when $m \ge 1$ bits are extracted from a transform coefficient, are

$$b_k = Q^{-1} \left(Q(b_0) \cdot \left(1 - \frac{k}{2^m} \right) + Q(b_{2^m}) \cdot \frac{k}{2^m} \right) \quad \text{for } k = 1, 2, \dots, (2^m - 1)$$
(2)

> Consider any transform coefficient realization T = t that is on the quantization boundary

 \Rightarrow Error probability is 0.5, so one cannot guarantee secrecy for all devices!! \Rightarrow Current PUF products only provide security guarantees for the average over all devices, so millions of intelligent IoT devices are likely vulnerable to malicious attacks!! > Model transform coefficients T as, e.g., **truncated Gaussian** distributions. Then, quantization boundaries, when $m \ge 1$ bits are extracted from a transform coefficient, are

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⇒ Current PUF products only provide security guarantees for the average over all devices, so millions of intelligent IoT devices are likely vulnerable to malicious attacks!!

We propose to eliminate realizations

$$\bar{t} \in ((b_k - \delta/2), (b_k + \delta/2)]$$
(3)

for all transform coefficients and for some fixed $\delta \ge 0$ that is called the **Quality-of-Security-Service (QoSS) parameter** for all PUF outputs that are used for secure and private device authentication.

Denote the ratio of eliminated realizations vs. all realizations as

$$\gamma(\delta) = \frac{\sum_{k=1}^{(2^m-1)} \left(Q\left(b_k - \frac{\delta}{2}\right) - Q\left(b_k + \frac{\delta}{2}\right) \right)}{Q(b_0) - Q(b_{2^m})}.$$
 (4)

> The percentage of realizations \bar{t} used for secret key agreement, i.e., manufacturing yield, is

$$\beta(\delta) = 100 \times (1 - \gamma(\delta)) \tag{5}$$

which decreases for increasing QoSS parameter δ \Rightarrow Stringent trade-off to be optimized!!

- Define a reliability metric called correctness probability P_c that measures the probability that all bits are correct.
- > $\mathbf{P_c}$ increases for increasing δ .

PUF Results from an RO Dataset

- > $P_c(\delta)$ vs. $\beta(\delta)$ when a selected low-complexity transform with *m*-bit uniform quantization is applied to 16×16 RO arrays.
- > We have $\beta = 100$ and P_c at its minimum when $\delta = 0$.



> Optimal Error-Correcting Code Designs for PUFs

- We propose binning-based code constructions that are Pareto optimal and improve on all existing methods [GIS+'19].
- Polar codes designed for RO and SRAM PUFs achieve rate tuples that cannot be achieved by existing methods.
- Significant performance improvements are illustrated for
 - Multiple PUF measurements [GKS'15, GK'18].
 - Adaptive PUF measurements [KGS+'16, GKS+'18],
 - Multiple PUF enrollments (uses) [KGW'18],
 - Multiple rounds of communication [GGK'18], etc.

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Problem Formulation – Generated-Secret (GS) Model



► The Shannon entropy measures the uncertainty, i.e.,

$$H(X) = -\sum_{x \in \mathcal{X}} P_X(x) \log(P_X(x)).$$

- > Mutual information measures statistical dependency, i.e., I(X;Y) = H(X) H(X|Y).
- Note that min-entropy is relevant to but different from the Shannon entropy, and we can discuss why the Shannon entropy is the right metric for secret key generation from PUFs/biometrics with nested codes!

Definition

A key-leakage-storage tuple (R_s, R_ℓ, R_w) is achievable if, given any $\epsilon > 0$, there is some $n \geq 1$ for which $\displaystyle \frac{\pmb{R_s}}{\pmb{R_s}} = \frac{\log |\mathcal{S}|}{2}$ and $\Pr[\widehat{S} \neq S] < \epsilon$ (reliability) (6)
$$\begin{split} &\frac{1}{n}I(S;W)\leq\epsilon\\ &\frac{1}{n}H(S){\geq} \textbf{\textit{R}}_{s}-\epsilon\\ &\frac{1}{n}I(X^{n};W){\leq} \textbf{\textit{R}}_{\ell}+\epsilon\\ &\frac{1}{n}\log\left|\mathcal{W}\right|{\leq} \textbf{\textit{R}}_{w}+\epsilon \end{split}$$
(7)(secrecy leakage) (8)(key uniformity) (9) (privacy leakage) (public storage). (10)

Theorem (Ignatenko and Willems'09)

The key-leakage-storage region for the GS model is

$$\mathcal{R}_{gs} = \bigcup_{P_{U|X}} \left\{ (R_s, R_\ell, R_w) : 0 \le R_s \le I(U; Y), \\ \begin{array}{c} R_\ell \ge I(U; X) - I(U; Y), \\ R_w \ge I(U; X) - I(U; Y) & \text{for} \\ P_{UXY} = P_{U|X} P_X P_{Y|X} \end{array} \right\}.$$
(11)

- Code-offset fuzzy extractors (COFE) [Dodis et al.'08] for the GS model,
- Fuzzy-commitment scheme (FCS) [Juels and Wattenberg'99] for the chosen-secret (CS) model,
- Syndrome-based Polar Code Construction [Chen et al.'17] for the GS model.

 COFE and FCS result in a storage rate of 1 bit/symbol since they apply one-time padding.

Syndrome-based polar code construction

- improved on existing methods because it is a Slepian-Wolf coding (i.e., lossless compression) construction,
- > achieves only a single point on the region \mathcal{R}_{gs} boundary.

We now show that our Wyner-Ziv (WZ)-coding (i.e., lossy compression) constructions [Shamai et al.'98, Korada et al.'10] are Pareto-optimal.

Assume

►
$$X^n \sim \text{Bern}^n\left(\frac{1}{2}\right)$$
, i.e., $\Pr[X_i = 1] = \Pr[X_i = 0] = 0.5$ for all $i = 1, 2, ..., n$.

P_{Y|X} is a binary symmetric channel (BSC) with crossover probability p_A, i.e., Pr[Y ≠ X] = p_A.

Choose uniformly at random full-rank parity-check matrices H₁, H₂, and H as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}$$
(12)

- ▶ $\mathbf{H}_1 \in \{0,1\}^{m_1 \times n}$ defines a binary linear code C_1 with parameters $(n, n-m_1)$, meaning that the codewords have size nand there are 2^{n-m_1} codewords.
- ▶ $\mathbf{H} \in \{0,1\}^{(m_1+m_2) \times n}$ defines a binary linear code C with generator matrix \mathbf{G} and parameters $(n, n-m_1-m_2)$,

$$\blacktriangleright$$
 Codes are **nested**, i.e., $\mathcal{C} \subseteq \mathcal{C}_1$

➤ Impose the conditions, for some q ∈ [0, 0.5] and δ > 0 (not to be confused with the QoSS parameter),

$$\frac{k_1}{n} \triangleq \frac{n - m_1}{n} = 1 - H_b(q) - \delta,$$
(13)
$$\frac{k}{n} \triangleq \frac{n - m_1 - m_2}{n} = 1 - H_b(q * p_A) - 2\delta$$
(14)

where

►
$$H_b(q) = -q \log q - (1-q) \log(1-q)$$

► $q * p_A = q(1-p_A) + (1-q)p_A.$

Problem Formulation (Recall)



Enrollment:

▶ Observe X^n and find the codeword $X^n_a \in C_1$ such that

$$X_q^n = \operatorname*{arg\,min}_{C^n \in \mathcal{C}_1} d_H(X^n, C^n) \tag{15}$$

where $d_H(\cdot)$ is the Hamming distance,

▶ Error sequence $X^n \oplus X^n_q \triangleq E^n_q \sim \text{Bern}^n(q)$ when $n \to \infty$,

> Assign $W = X_q^n \mathbf{H}_2^T$ as helper data since $X_q^n \mathbf{H}^T = [0 \ W]$,

- Sum X_q^n with the sequence L_W^n that is in the same coset as X_q^n and that has the minimum Hamming weight. The sum is $X_q^n \oplus L_W^n = X_c^n \in C$,
- > Assign the secret key S such that $X_c^n = S\mathbf{G}$,

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> Codewords in blue and green belong to C_1 ,

Codewords in green belong to C. W = 0W = 1 $W = 2^{m_2} - 1$. . . $C_0^n = L_0^n = 00..0$ S = 0 $L_1^n = 00..1$ $L_{2^{m_2}-1}^n = 01..1$ · S = s $\overline{C_s^n}$ $C_s^n \oplus L_1^n$ $\overline{C^n_s} \oplus L^n_{2^{m_2}-1}$ $S = 2^k - 1$ $C_{2^{k}-1}^{n}$ $C_{2^k-1}^n \oplus L_1^n$ $C_{2^{k}-1}^{n} \oplus L_{2^{m}2-1}^{n}$

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Problem Formulation (Recall)



Reconstruction:

▶ The channel $P_{Y^n|X^n_a} \sim \text{Bern}^n(q * p_A)$ when $n \to \infty$,

 $\triangleright C$ can correct errors in $P_{Y^n|X_a^n}$ with high probability to estimate X_q^n ,

> \widehat{X}_q^n determines \widehat{S} .

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Polar Codes (used in the 5G wireless communications standard)

- ➤ A polar transform converts an input sequence Uⁿ with frozen and unfrozen bits to a codeword Xⁿ.
- ➤ Polar codes [Arıkan'08] rely on converting the physical channel Pⁿ_{Y|X} into virtual channels P_{YⁿUⁱ⁻¹|U_i}.

WZ Polar Code Construction (Cont'd)



- Key length 128 bits,
- > Block error probability $P_B = 10^{-6}$,
- ► $P_{Y|X} \sim \mathsf{BSC}(p_A = 0.15).$

Design nested polar codes in combination with successive cancellation list (SCL) decoders with list size 8.

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Rate-tuple Comparisons (Cont'd)



Conclusion

- We proved that security guarantees can be given (unlike some of the current PUF products) to every digital device with a PUF;
- Illustrated that by removing a small percentage of manufactured PUF circuits, it is possible to significantly simplify the error-correcting code design due to increased reliability;
- Conversely, we proved that without removing any PUF circuit output, no security guarantee per PUF;
- We illustrated that the proposed nested linear block code constructions are optimal for key extraction from PUFs;
- > We estimated the information leakage of nested polar codes by using neural estimators to showcase that practical PUF designs with our proposed code constructions leak only a negligible amount of information.

THANK YOU!

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