# SPAD-based QRNGs

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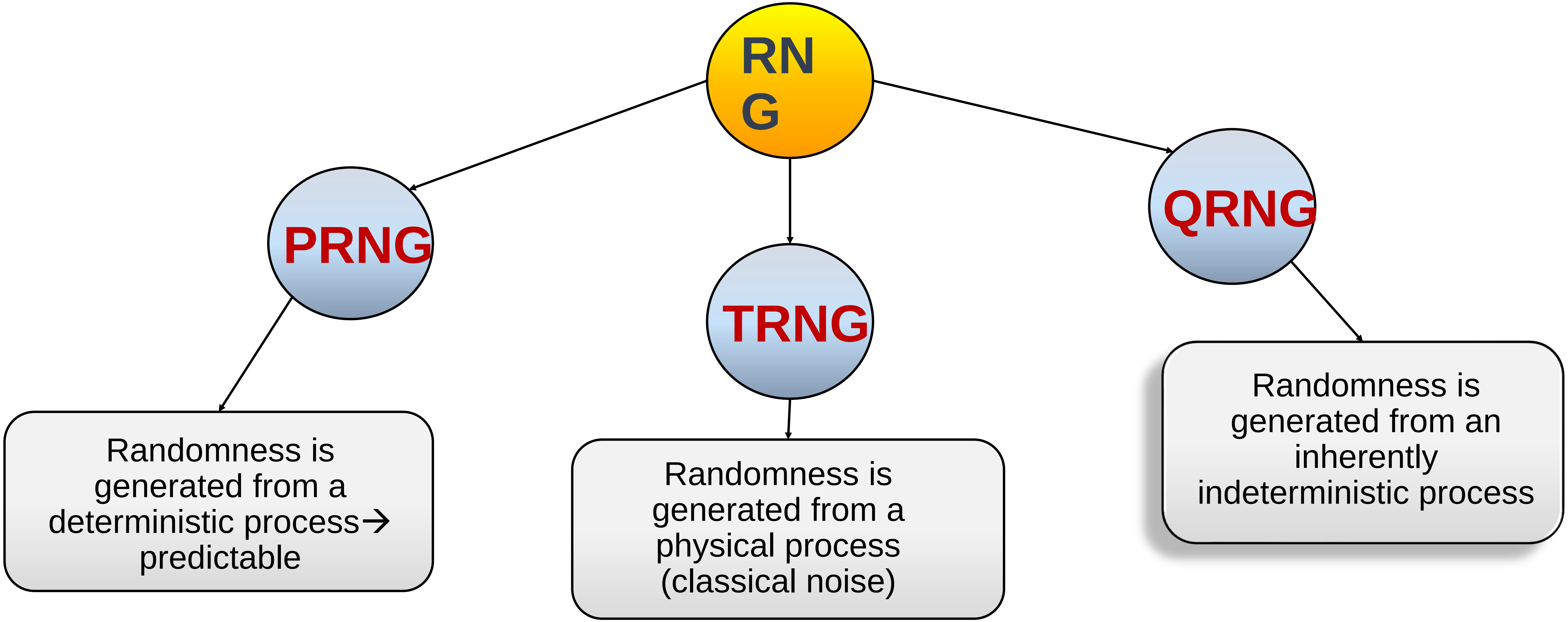
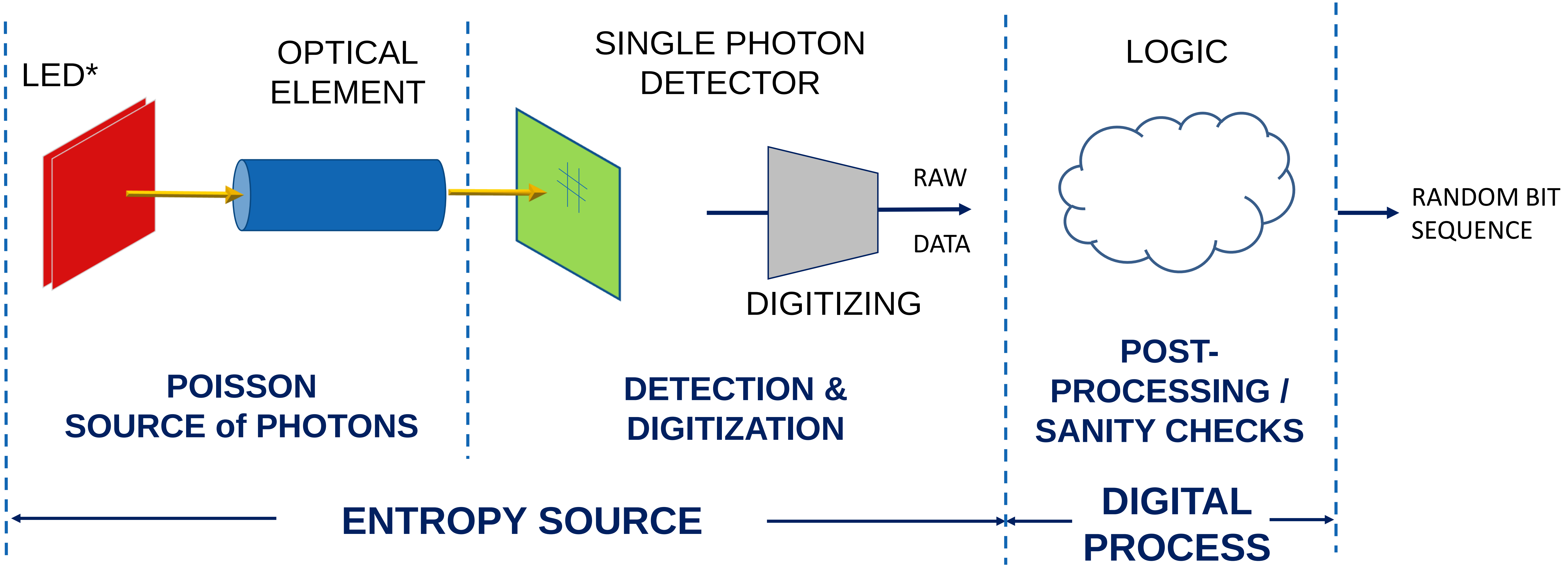
# QRNG based on SPAD
## Outline

- Introduction to QRNG based on SPAD
- Different QRNG approaches:
  - Based on photon counting
  - Based on the arrival time: single and multi-bit
  - Random FF
- QRNGs trend:
  - Monolithic SPAD
- Conclusion

# Context and motivation
## Quantum Random Number Generator



# SPAD-based QRNG
## Typical scheme

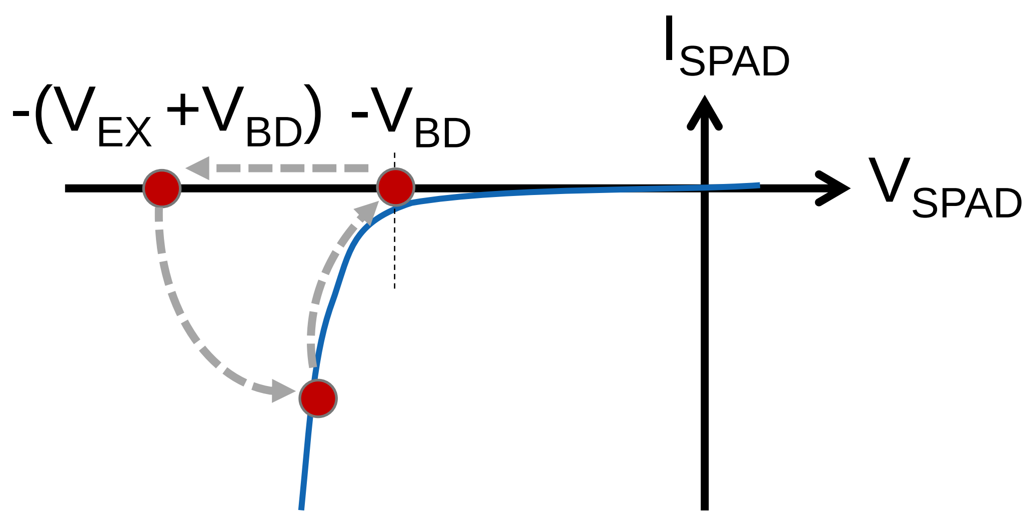



*An LED produces incoherent light by spontaneous emission which is essentially a random process. If operated at sufficiently low power, a LED emits photons which are virtually independent of each other

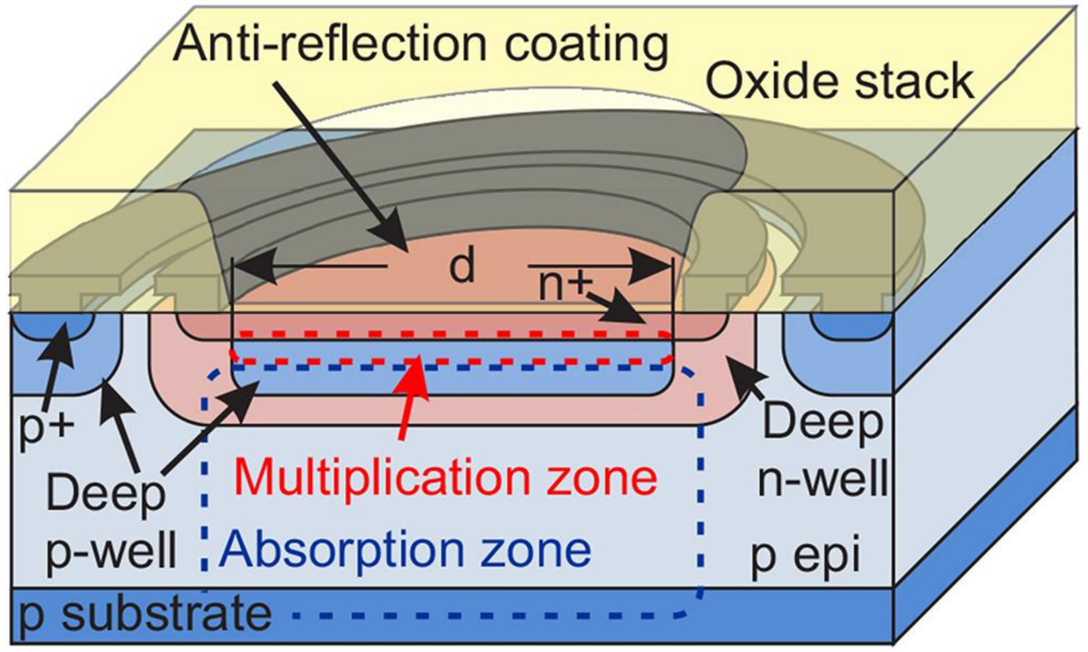
# SPAD-based QRNG
## Single Photon Avalanche Diode (SPAD)

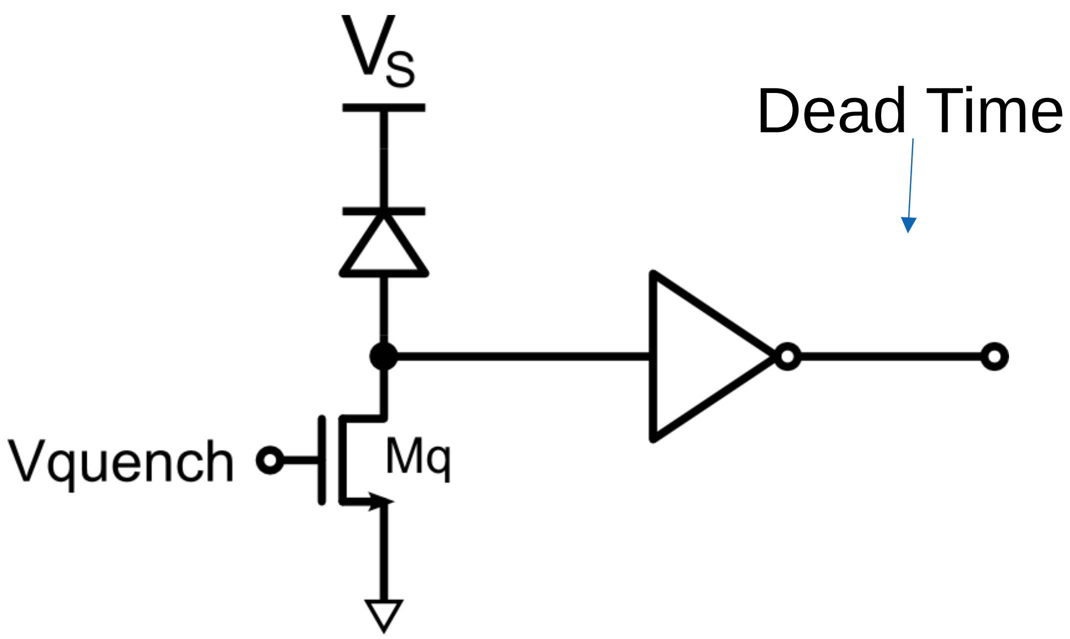
**SPAD (biased in Geiger mode) operation**

Avalanche
Quenching
Recharge

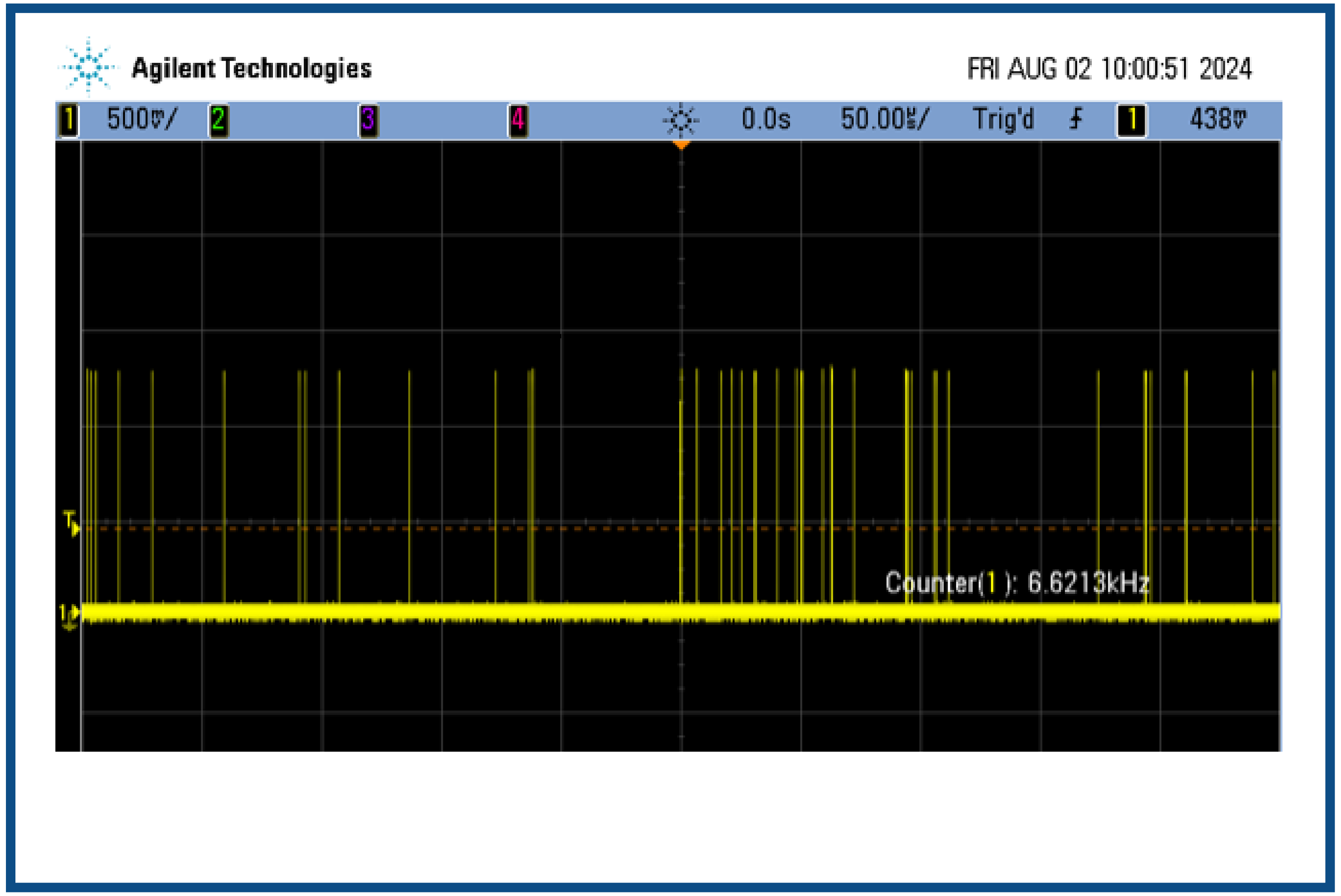


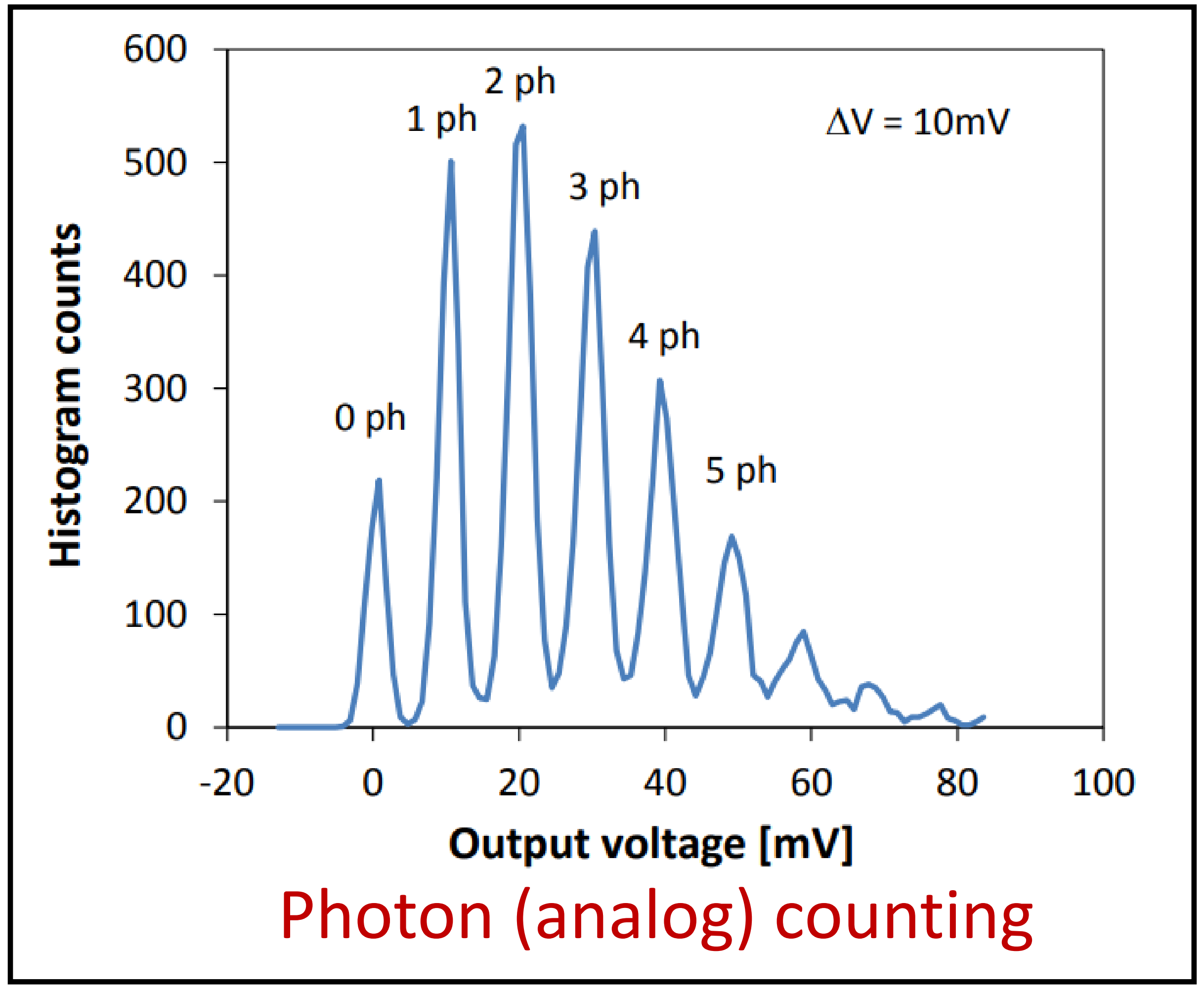

**Integration in a CMOS process**

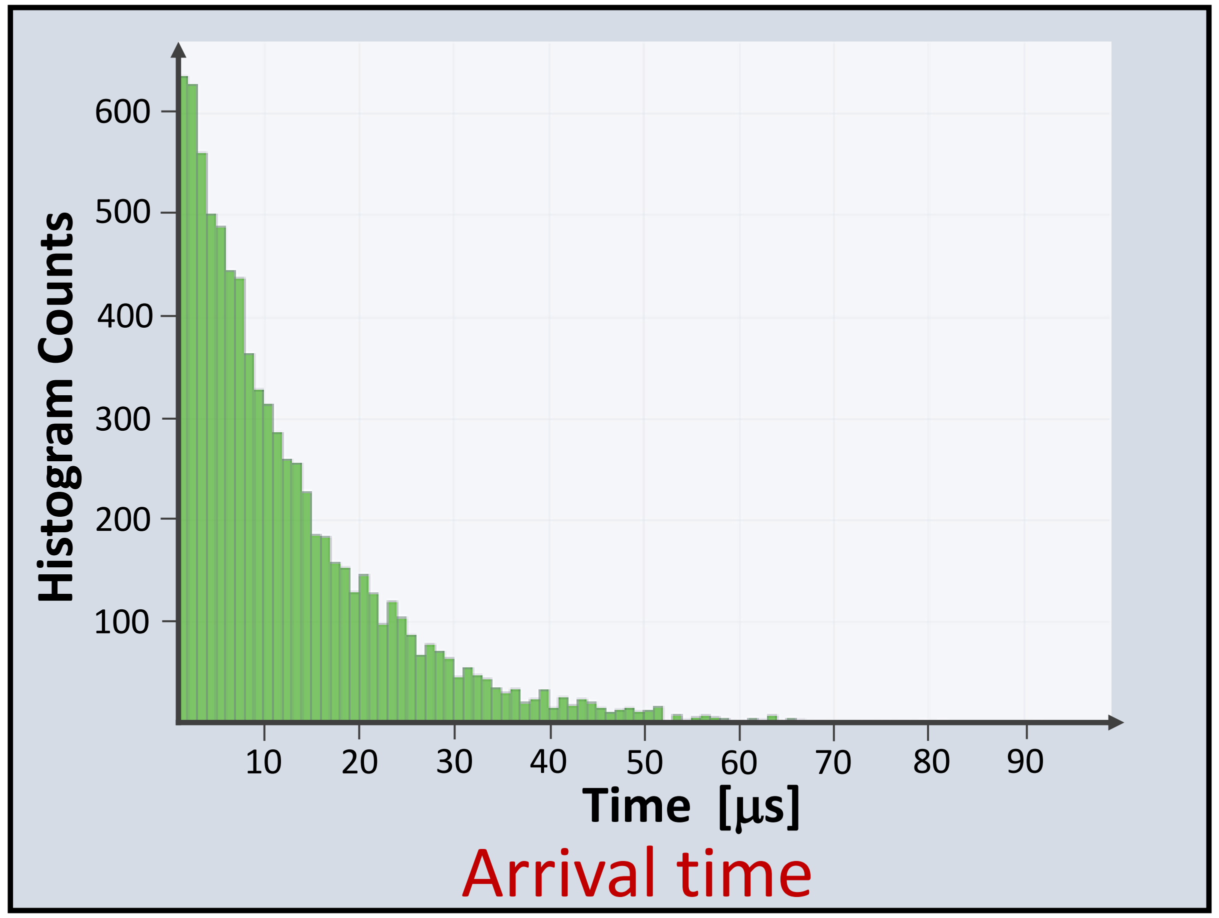


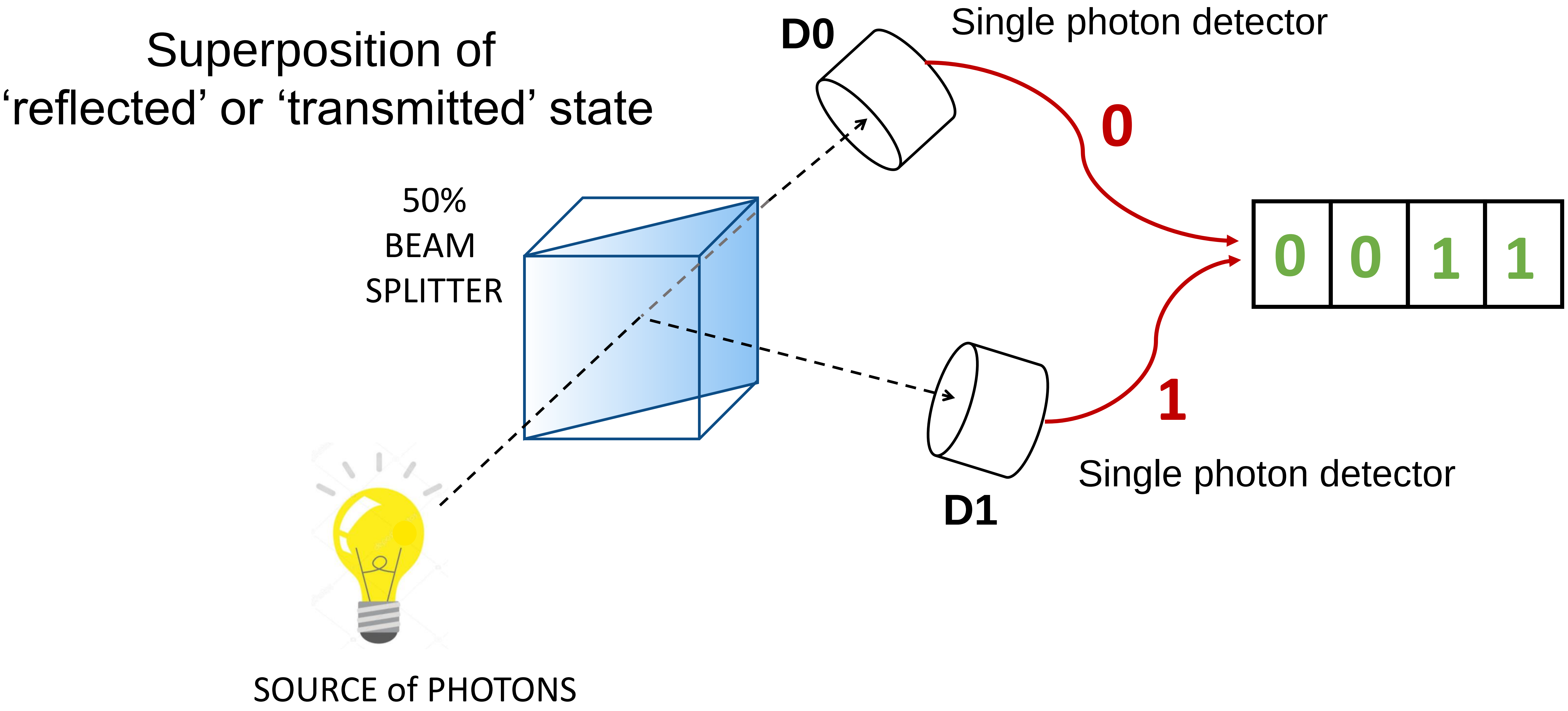
# SPAD-based QRNG
## SPAD output



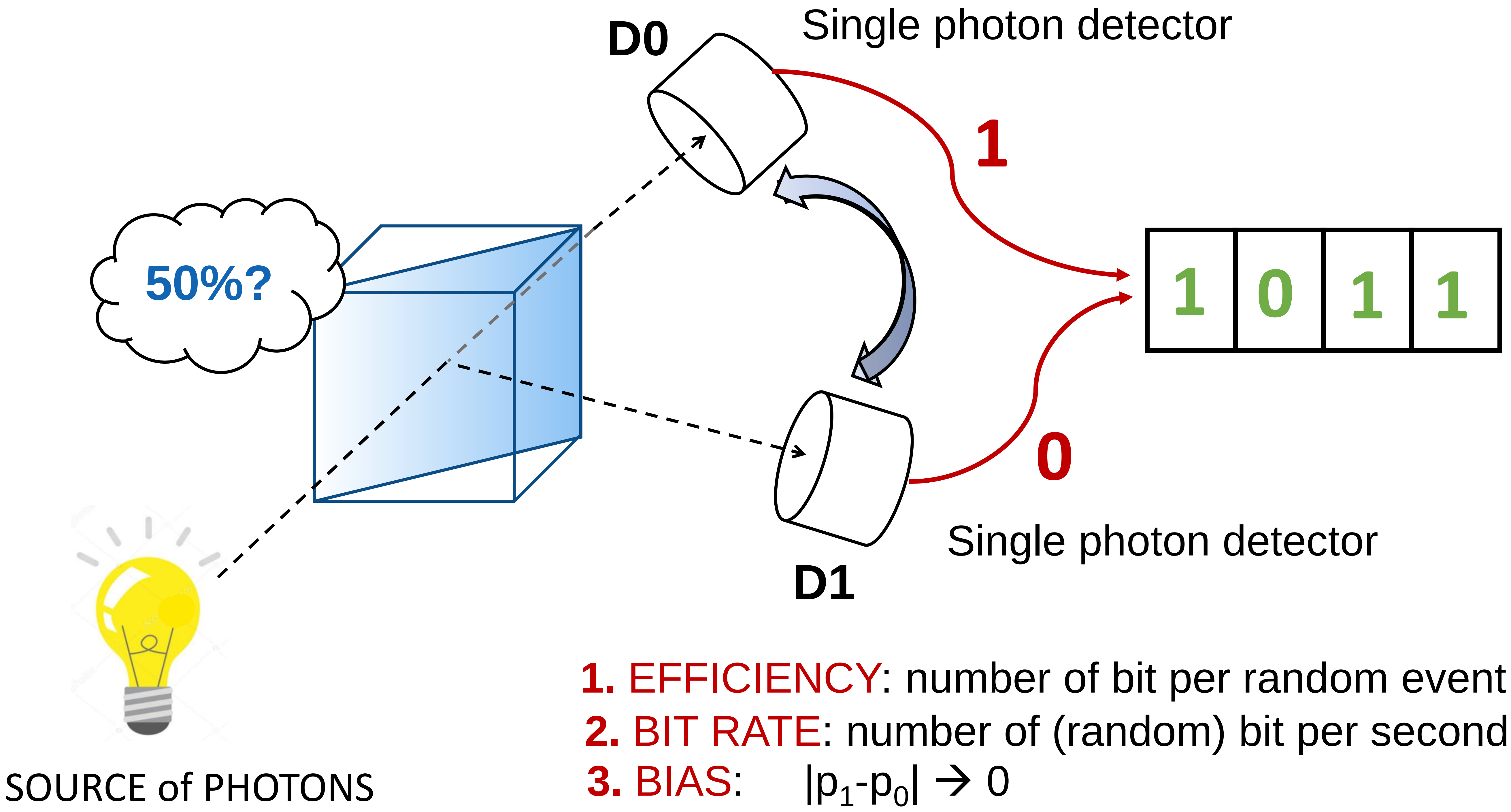



# Quantum Random Number Generator
## A prototypical QRNG

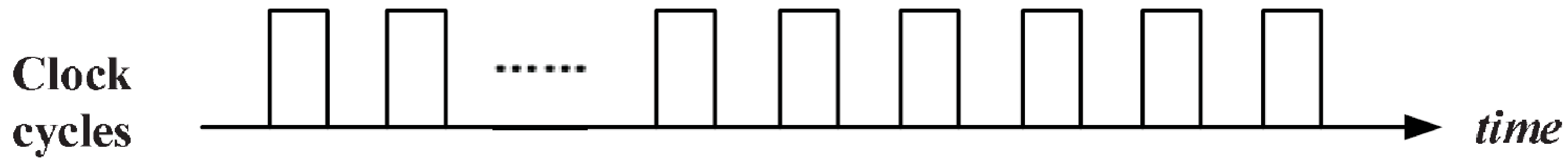


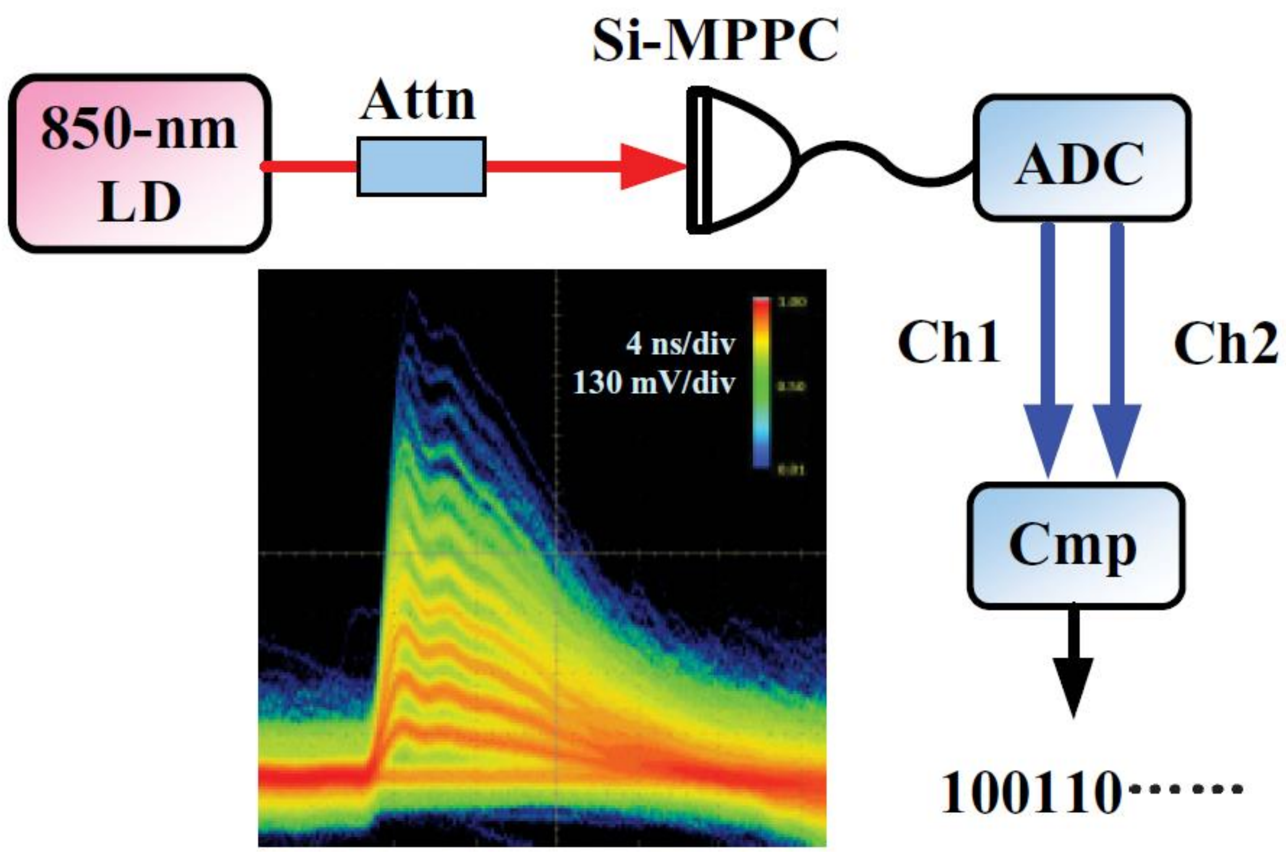
# Quantum Random Number Generator
## Main parameters




1. EFFICIENCY: number of bit per random event (detected ph)
2. BIT RATE: number of (random) bit per second
3. BIAS: $|p_1 - p_0| \rightarrow 0$
4. ENTROPY: $H(X) = -\sum_i p_i \log(p_i)$

# QRNG based on photon counting
## Single SPAD with external laser




$$P(N) = \frac{(\lambda T)^N e^{-\lambda T}}{N!}$$

The efficiency η of random bit generation is equal to

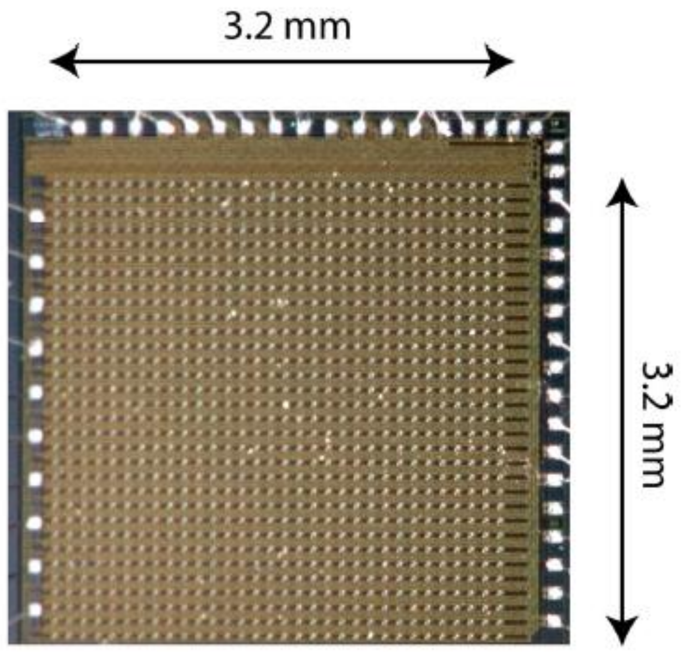
→ maximum 50% if a counter with N → ∞

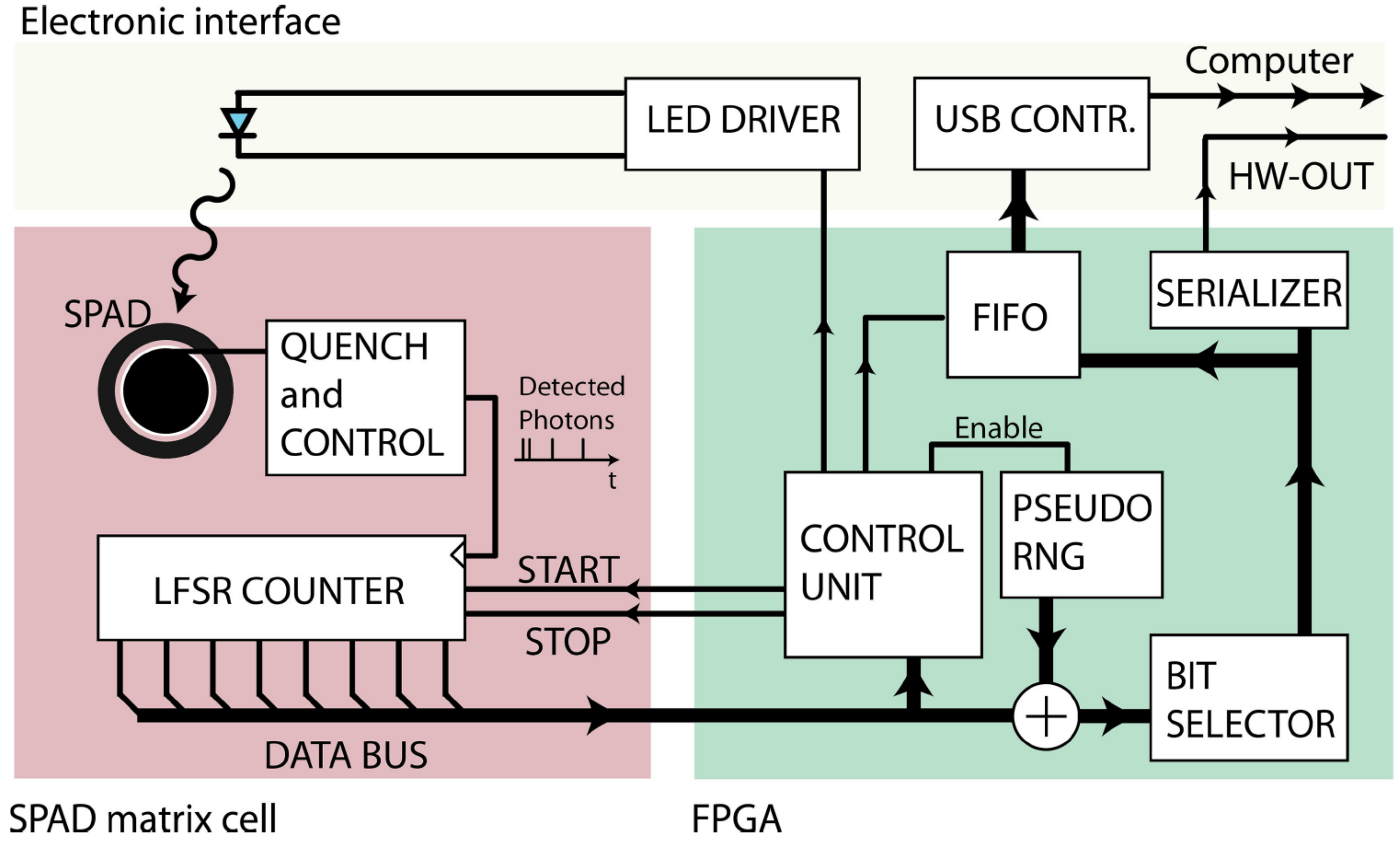
$$\eta = \frac{P(1) + P(0)}{2}$$

η is reduced because of the probability of $n_{i+1} = n_i$ → no random bit generation

*[Ren11]*

# QRNG based on Photon Counting
## QRNG by MPD


*[Tisa15]*

Up to 200 Mbps

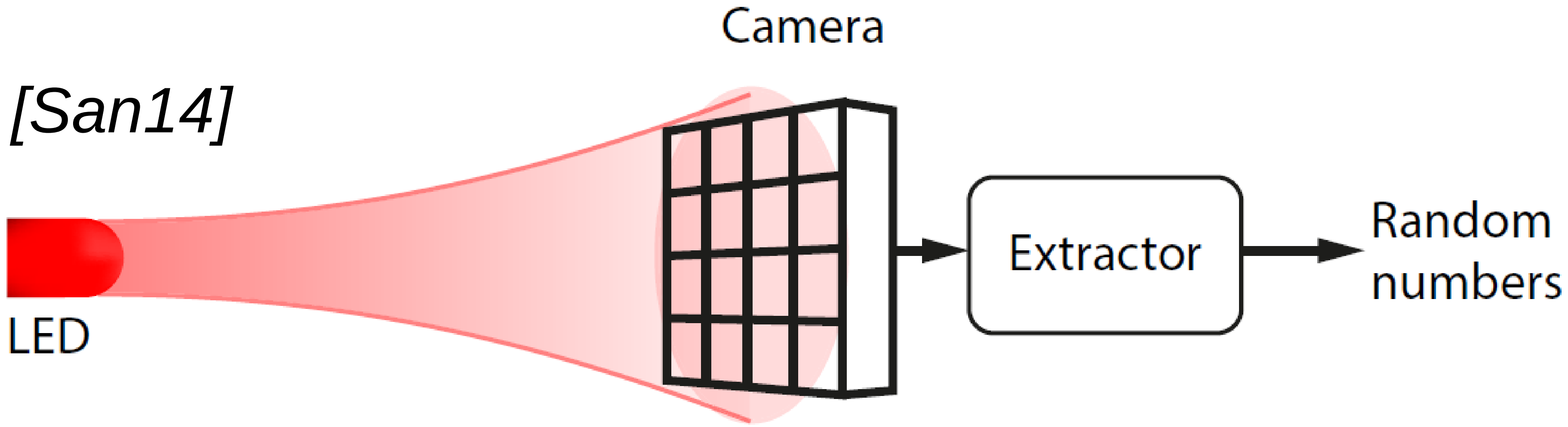
QRNG produced by MPD is based on an array of 32x32 SPAD cells connected to a photon counter. For high counts ($\lambda$>>10), the Poisson is ~ a Gaussian distribution with std equal to $\sqrt{\lambda}$. Choosing the LSB (parity bit) whitens the distribution. We can extend to more LSBs always guaranteeing a min entropy ~ $1/2\log_2(2\pi\lambda)$

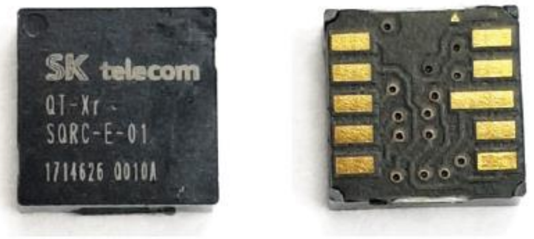
# QRNG based on Photon Counting
## IdQ product

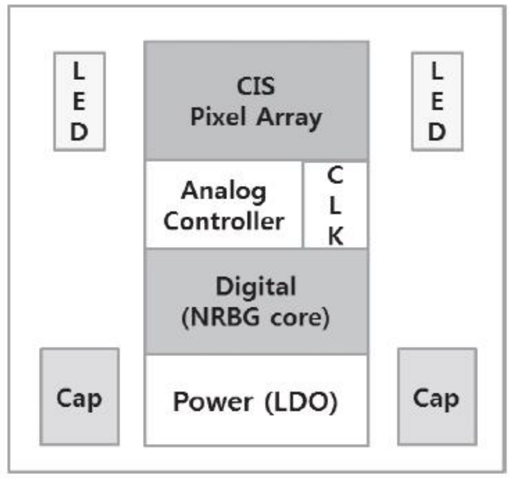
QRNG based on a standard digital camera.
The pixel value is dominated by shot noise and approximates well a Poission distribution

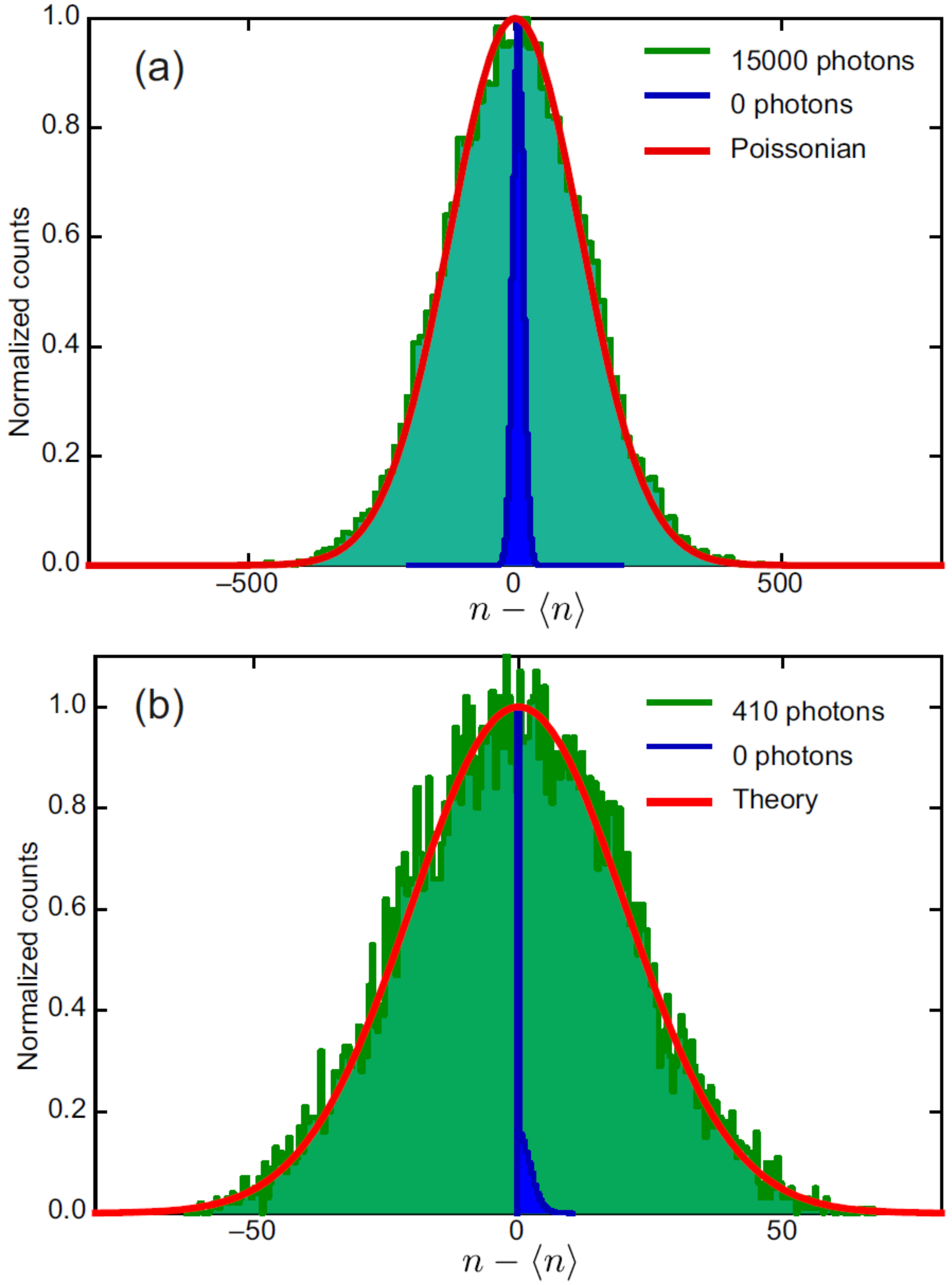
100 Mpixel → 3 bits per pixel → 0.3 up to 3 Gbps

*[San14]*

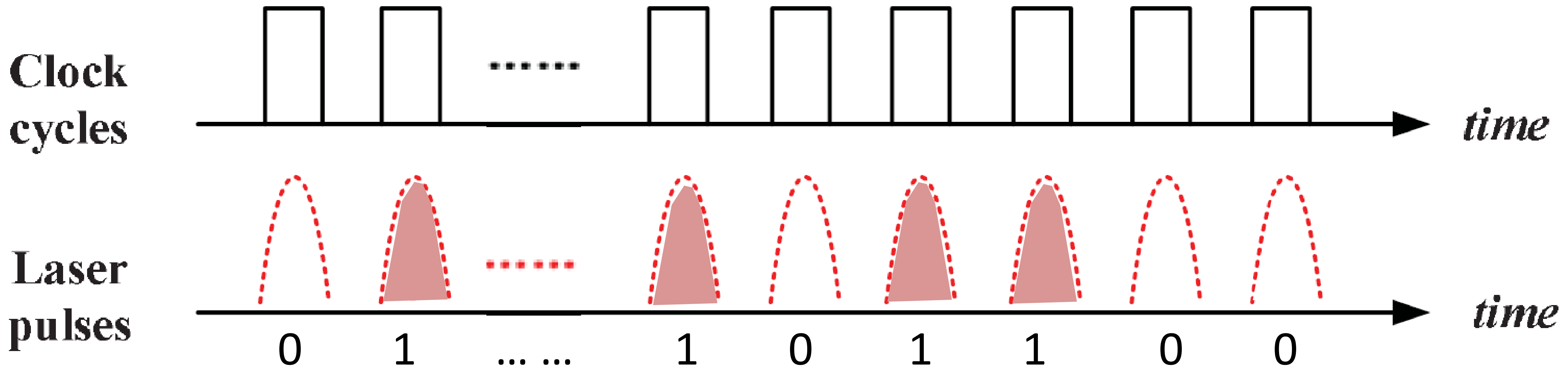


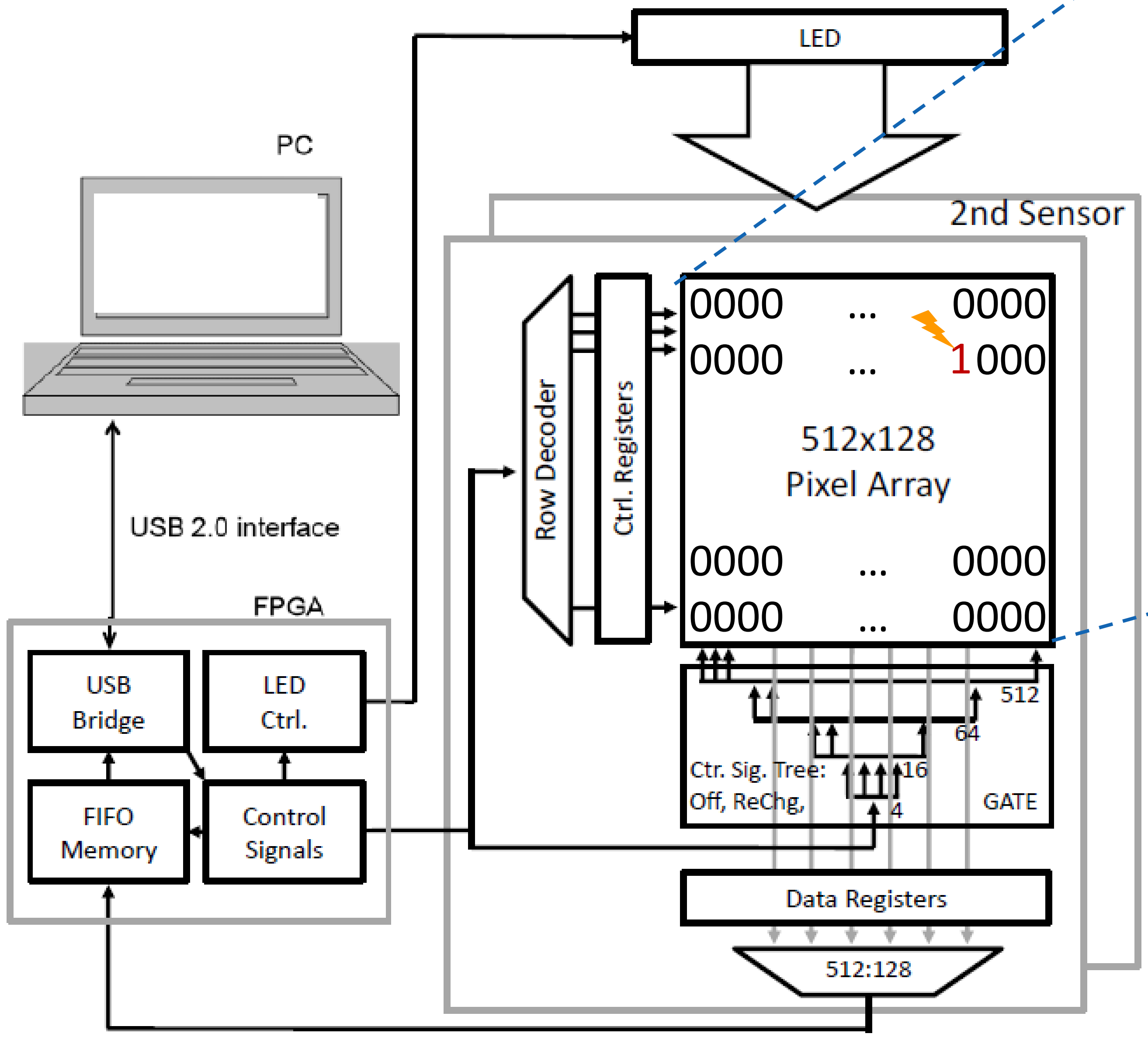



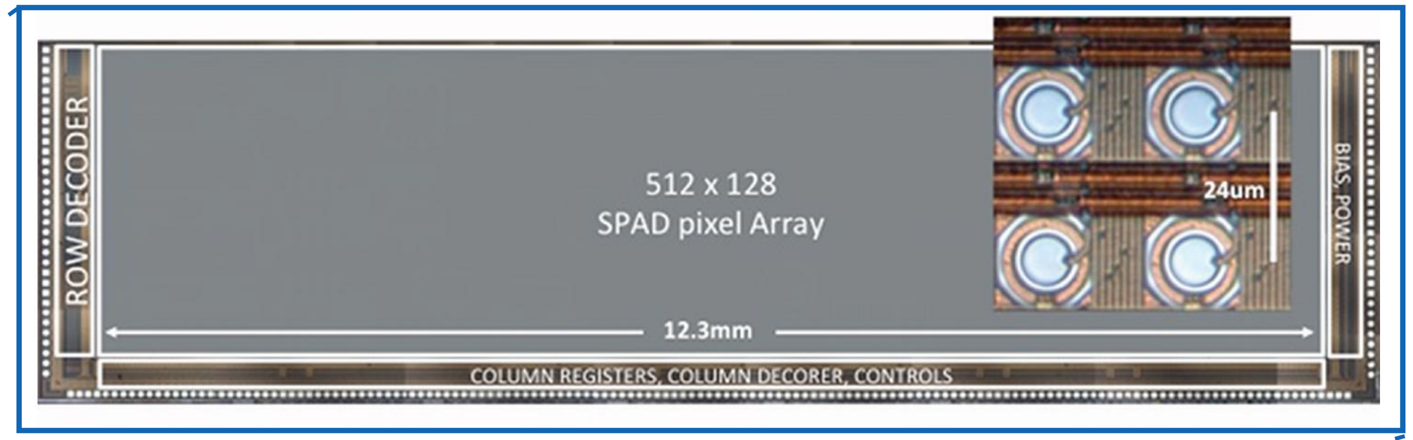
# QRNG based on photon counting
## Particular case when N=1

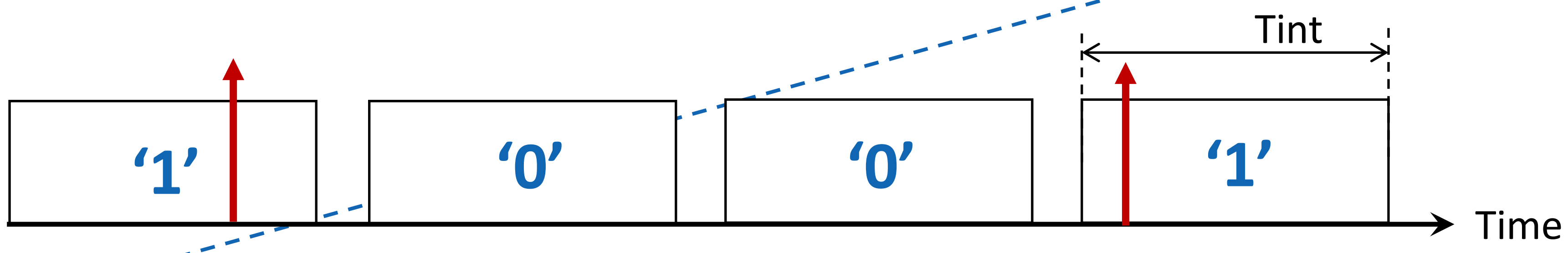



*[Wei18]*

# QRNG based on photon counting
## Fast (Gbps) binary single photon imager



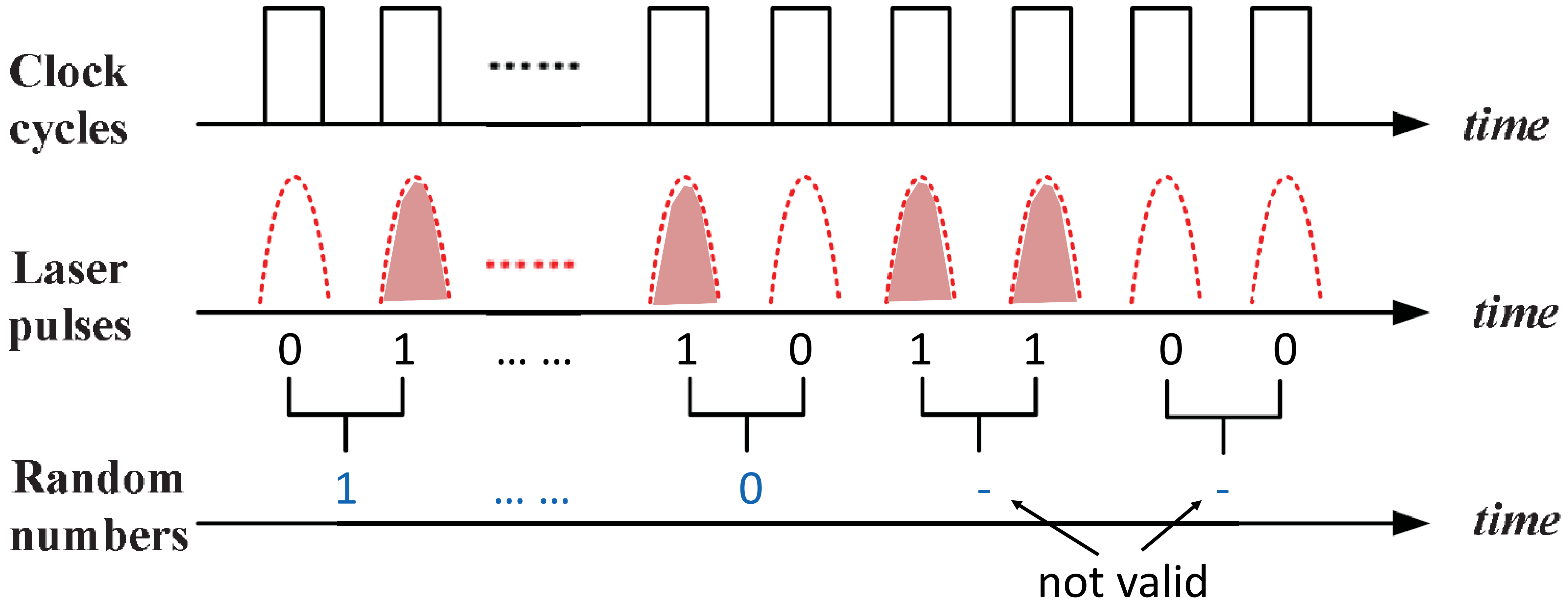




- Array of 512x128 pixels each having a SPAD
- Every cell of the array is reset to '0' and when it detects an event changes state ('0' → '1')

**5 Gb/s** maximum speed

*[Burri13]*

# QRNG based on photon counting
## Particular case when N=1 --> Von Neumann filter

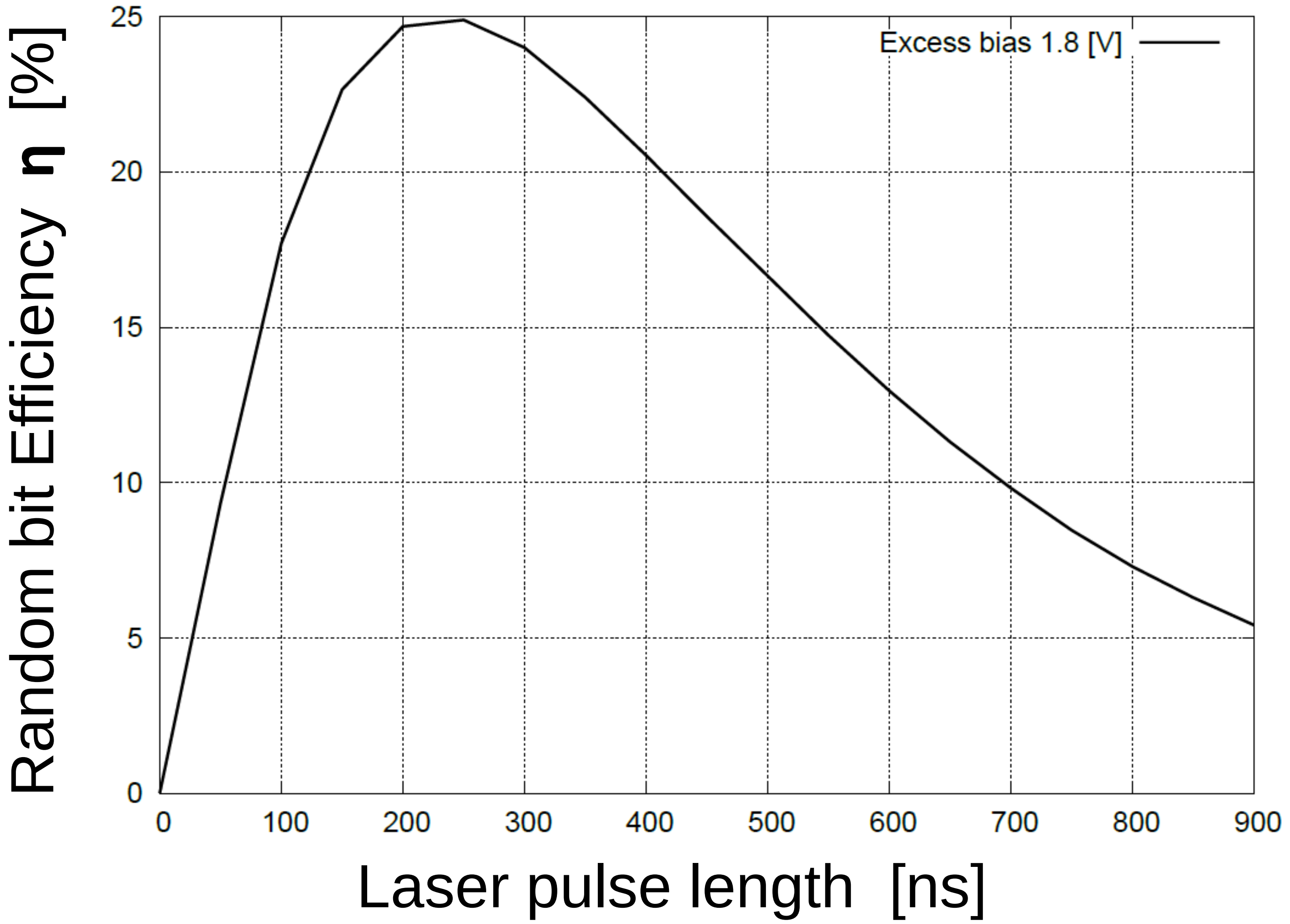



$$\eta = \frac{P('1') + P('0')}{2} = e^{-\lambda T} \cdot \left(1 - e^{-\lambda T}\right)$$

*[Wei18]*

Apply the Von Neumann filter to raw data: the maximum efficiency η=0.25 is reached at λT=ln2 ≈0.693 representing the value where probability of zeros and ones are equal

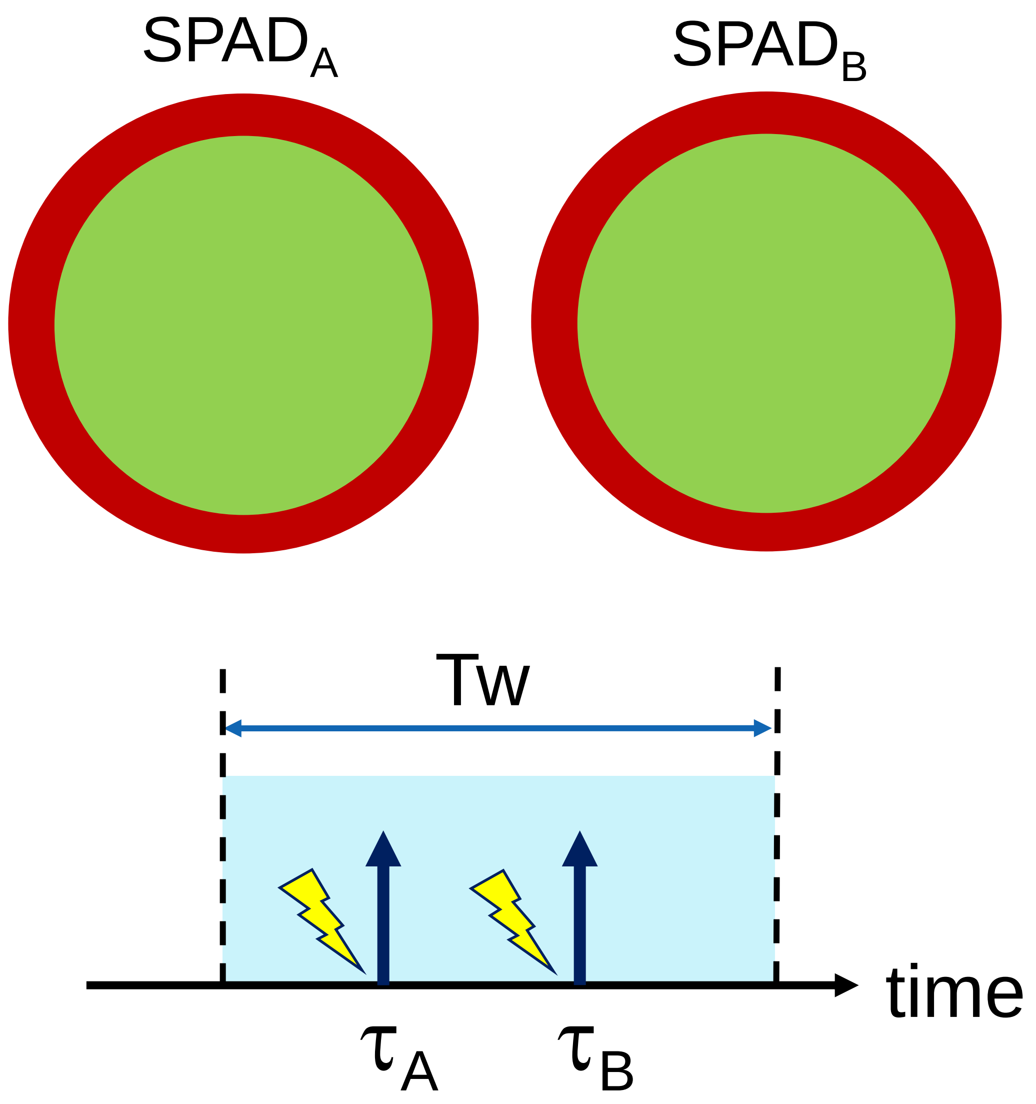
# QRNG based on photon counting
## Performance variation

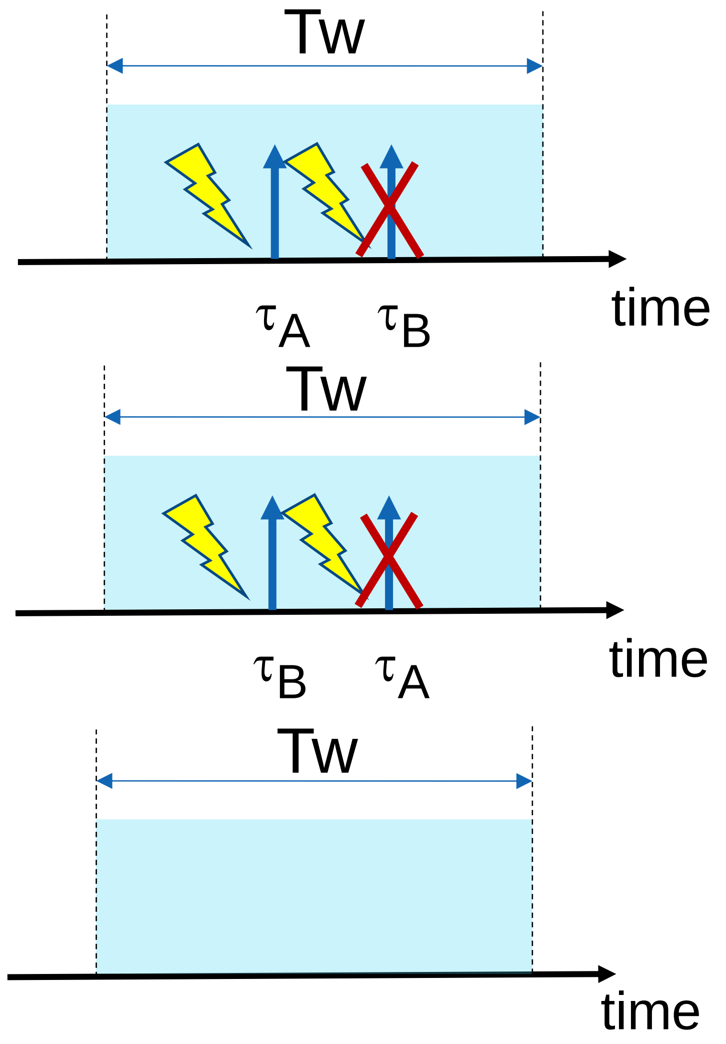



- The QRNG efficiency strongly depends on the flux of photons detected by every cell
- Difficult to guarantee a uniform behavior across the array
- May depends on aging or drift of the source of light

# QRNG based on the arrival time
## First detected photon

Let's consider a couple of SPAD with same size one close to the other

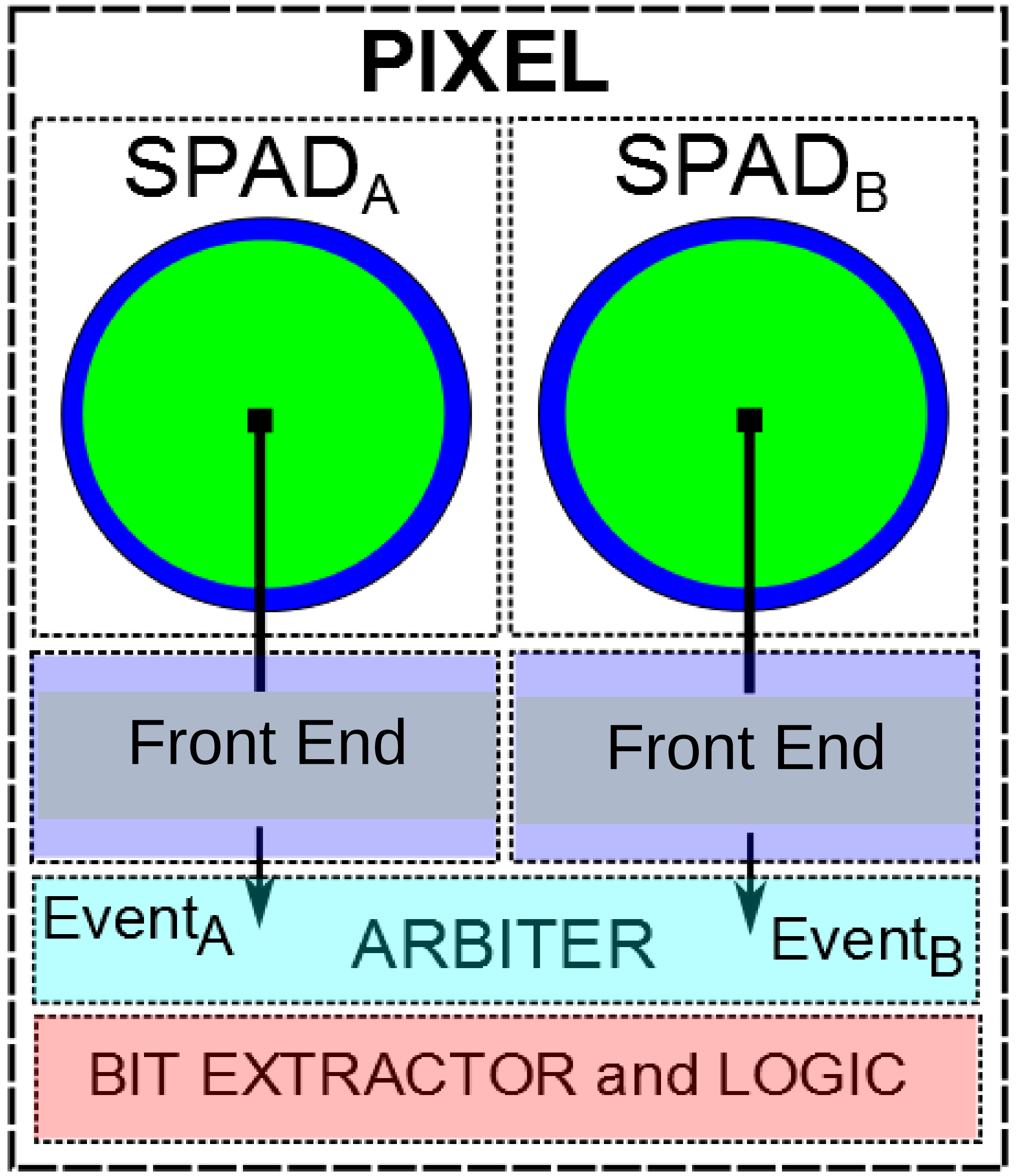



| **Output** | **When** |
|---|---|
| Bit '**1**' | $\tau_A < \tau_B$ |
| Bit '**0**' | $\tau_B < \tau_A$ |
| **No out** | **no event detection** |

*[Mas16]*
*[Xu18]*

# QRNG based on the arrival time
## First detected photon




- Competition between $SPAD_A$ and $SPAD_B$
- An arbiter has to identify the winner: $r_b = \begin{cases} '0' & if\ \tau_A > \tau_B \\ '1' & if\ \tau_A < \tau_B \end{cases}$

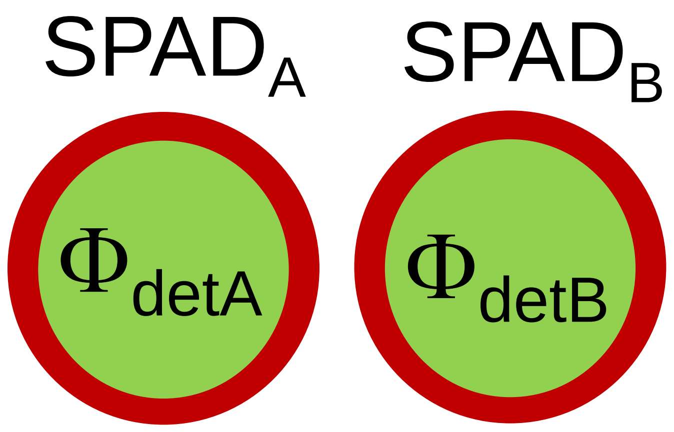
$$\begin{cases} P(\tau_A \leq t) = 1 - e^{-\Phi_{detA} t} \\ P(\tau_B \leq t) = 1 - e^{-\Phi_{detB} t} \end{cases}$$

$$\Rightarrow \boxed{P(\tau_A \leq \tau_B) = \frac{\Phi_{detA}}{\Phi_{detA} + \Phi_{detB}}}$$

If $\Phi_{detA} = \Phi_{detB}$ $\Rightarrow$ $P(\tau_A < \tau_B) = P(\tau_B < \tau_A) = 0.5$

# QRNG based on arrival time
## Source of bias: cell behaviour

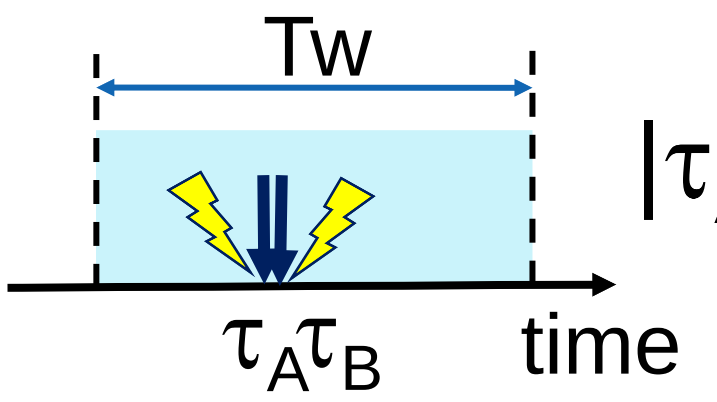
1.Mismatch between the two SPADs




- 1. Dark Count Rate (DCR
- 2. Detection Probability

$\Phi_{detA} \neq \Phi_{detB}$

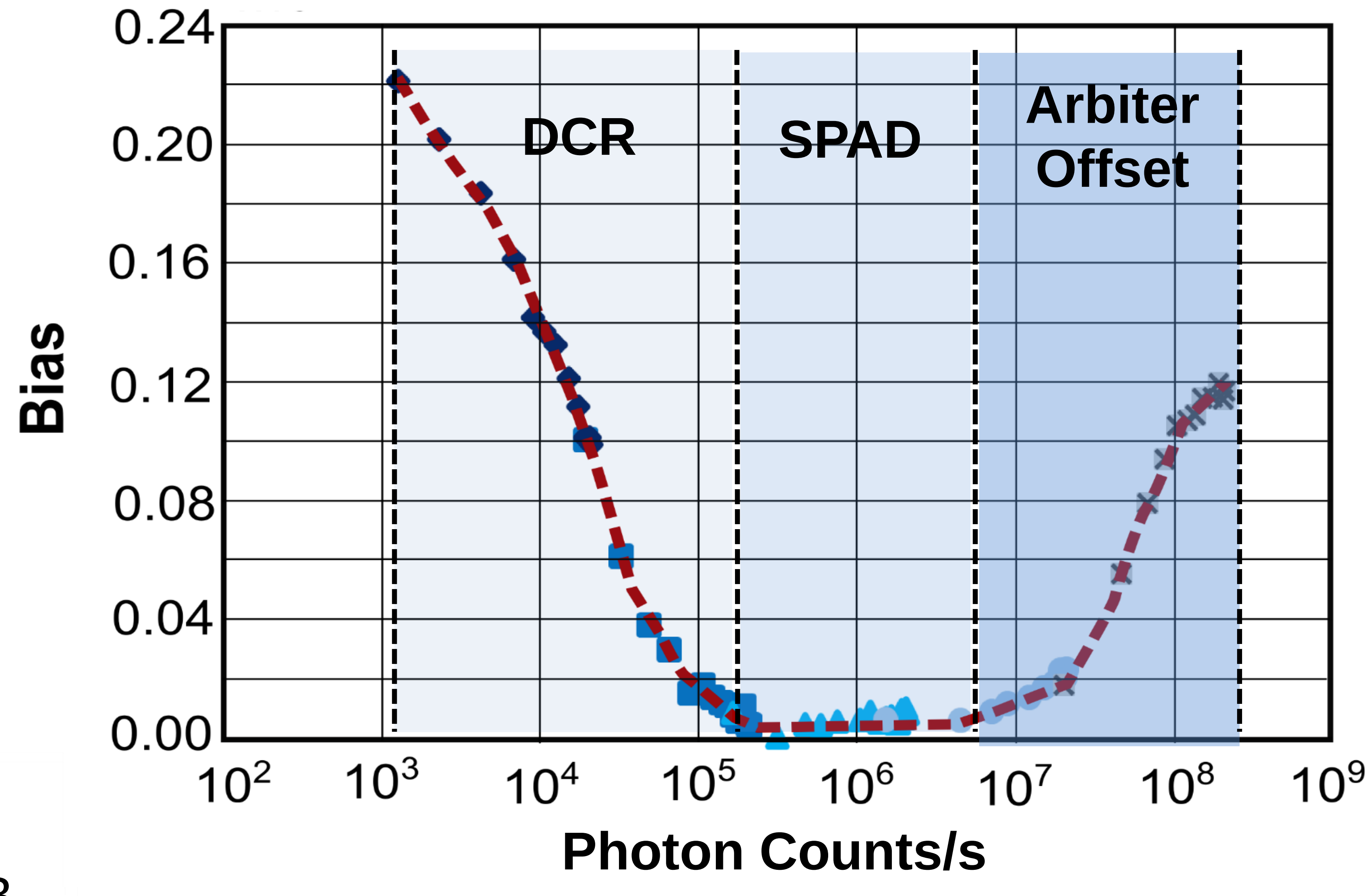
2. Circuit offset referred to the arbiter




$|\tau_A - \tau_B| < t_{OFF}$

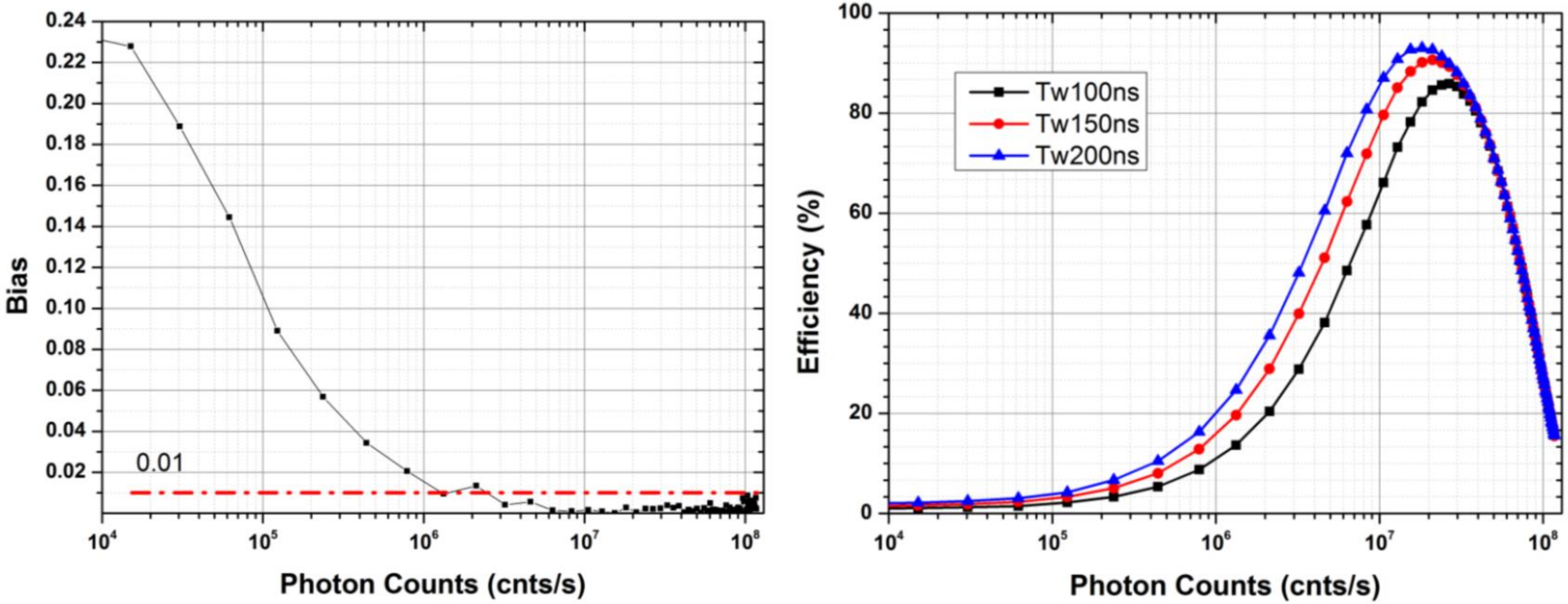
- 1.High $\Phi$
- 2. Crosstalk$_{A\leftrightarrow B}$

Mismatch due to the arbiter offset



# QRNG based on the arrival time
## Improved solution

- A circuit discards events that are too close in time:

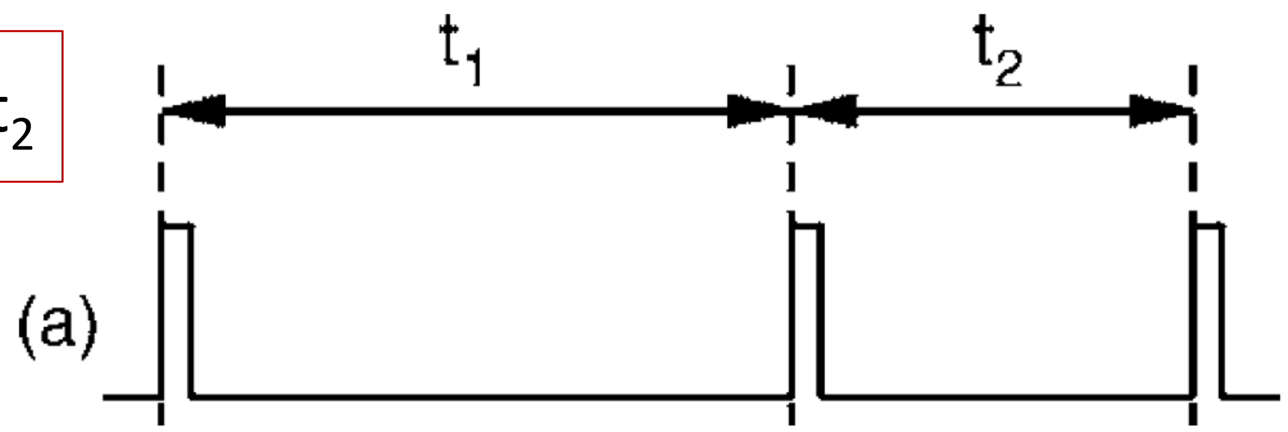



No use of TDC

Speed of 128 Mbps

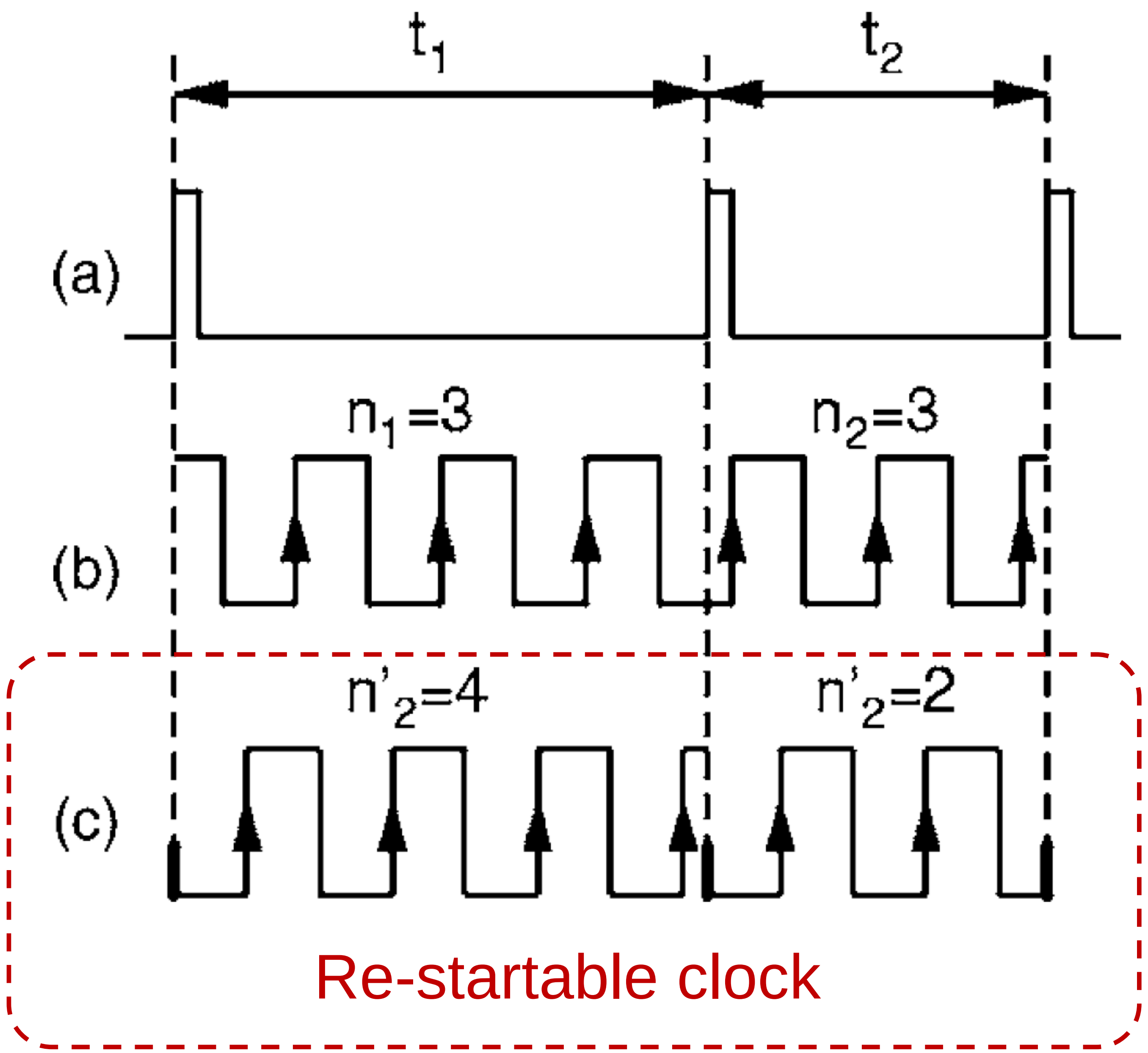
# QRNG based on the arrival time
## Time comparison

$t_1 < t_2$

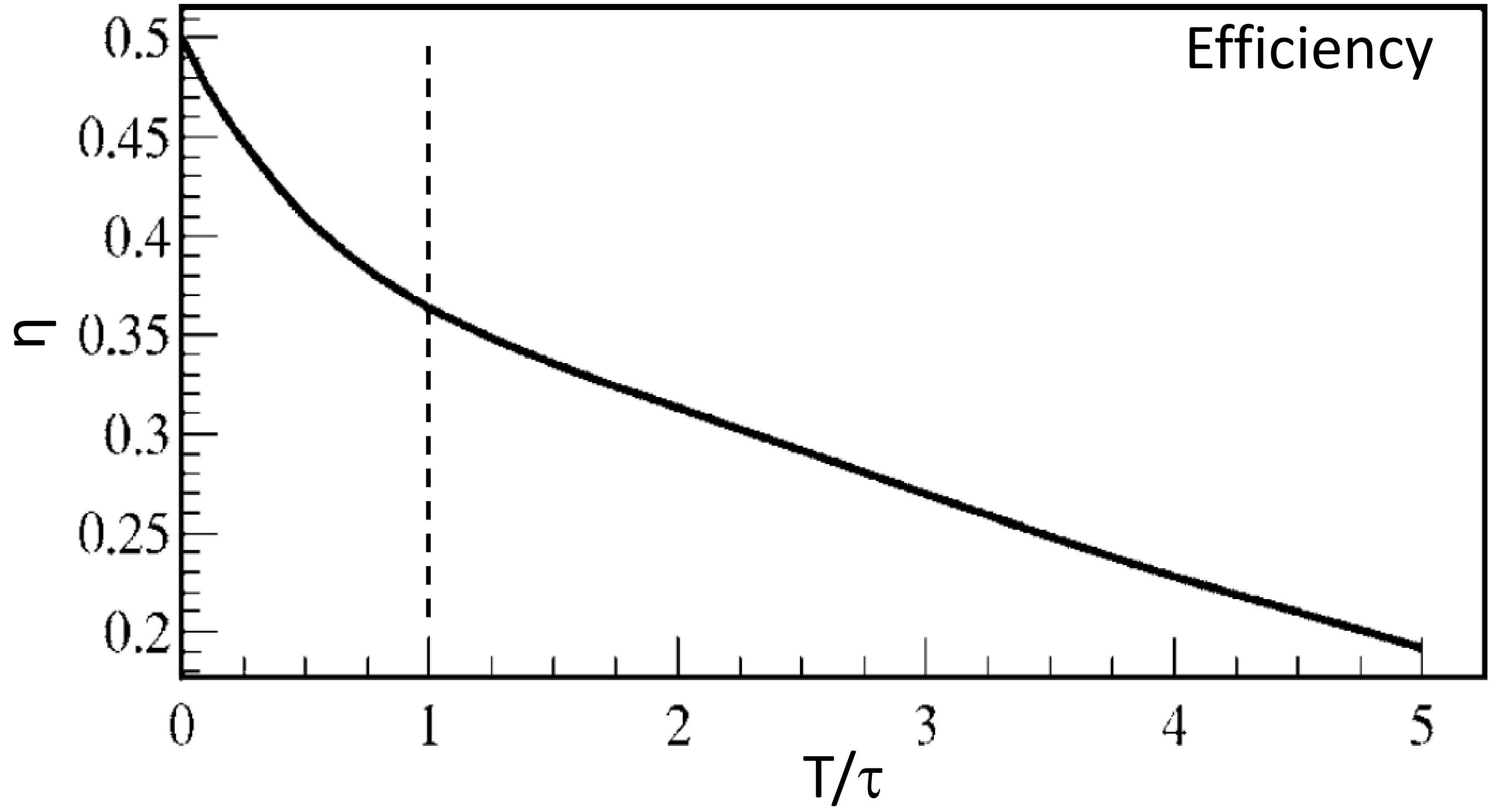



Using a frequency clock (1/T), we measure $n_1$ and $n_2$ → prob that $n_1 = n_2$ is $\neq 0$

# QRNG based on the arrival time
## Time comparison

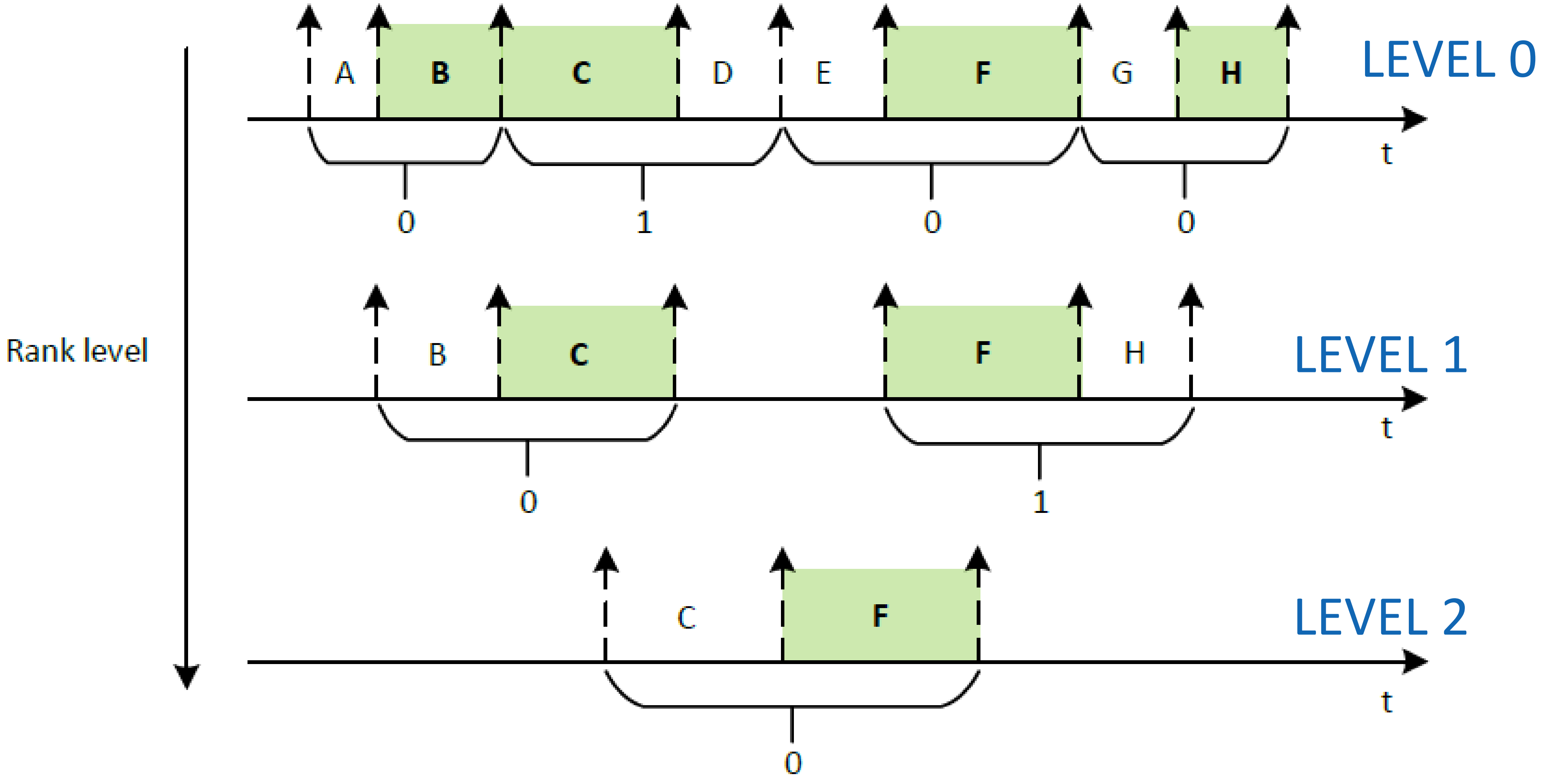



If $\tau$ is the average arrival time of events, the efficiency of the generator will be

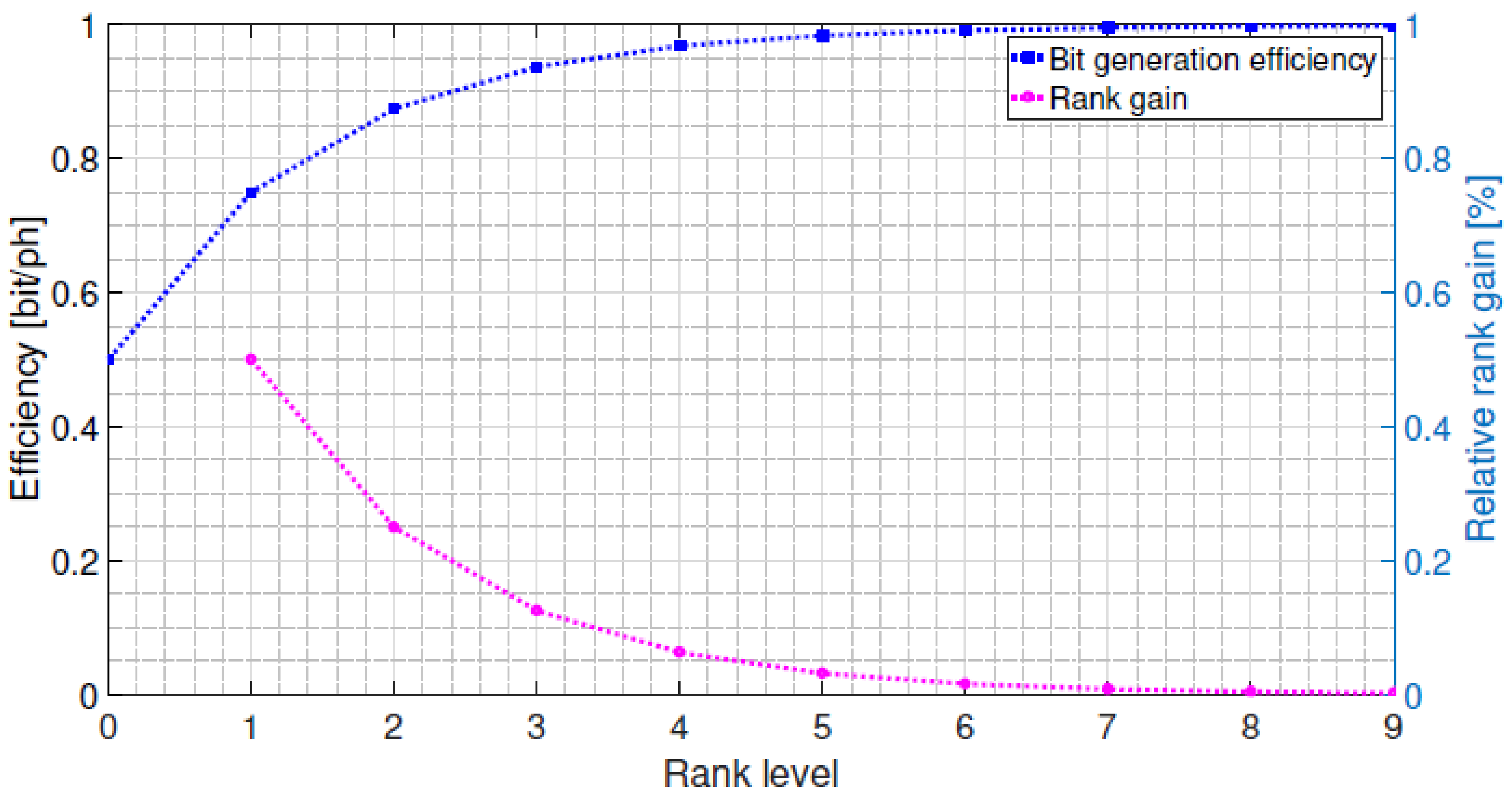


# QRNG based on arrival time
## Time comparison (extension)

Multiple ranks random number generation based on the arrival time



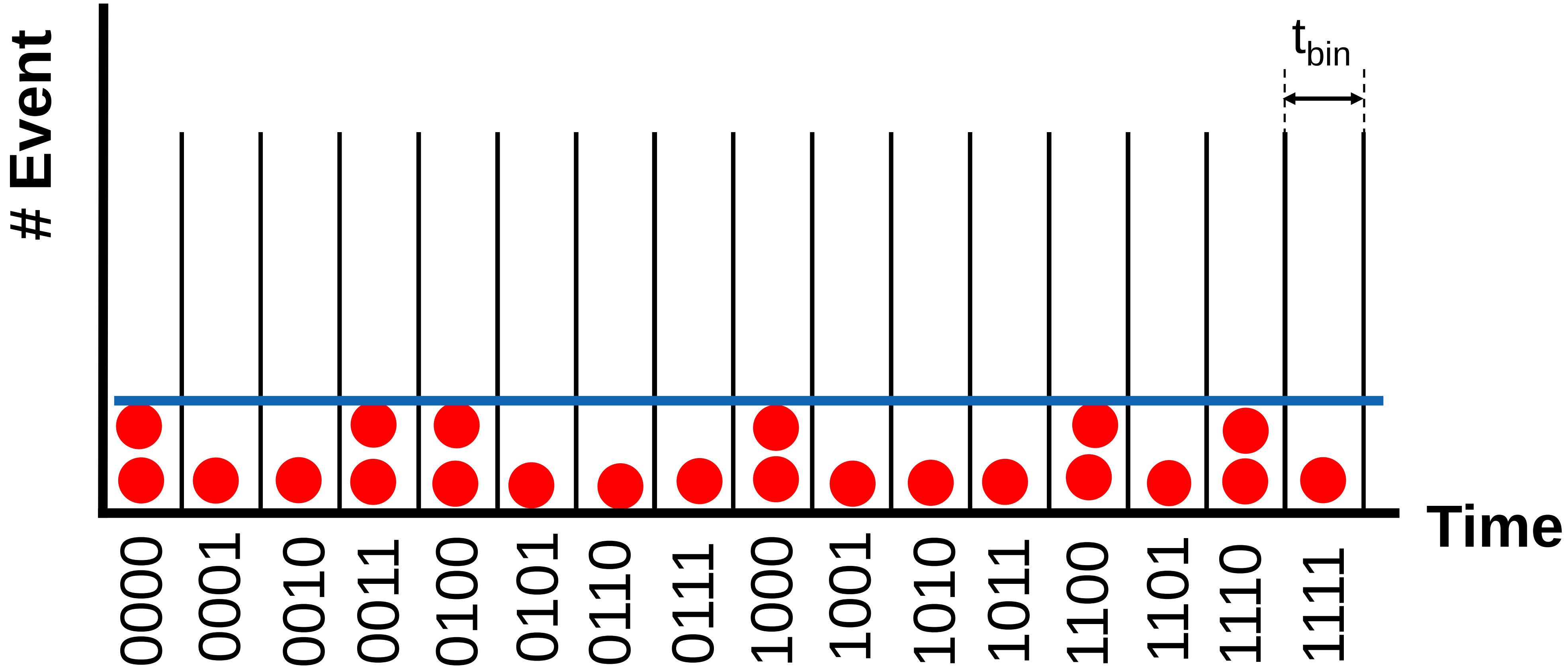

*[Ton19]*




Using 3 ranking levels we are exceeding 90% of efficiency

# QRNG based on arrival time
## Multi-bit generation

[Yan15]
[Bis17]
[Bis18]




λ = rate of events
$T_w$ = observation time window
$\lambda T_w$ = average events in Tw

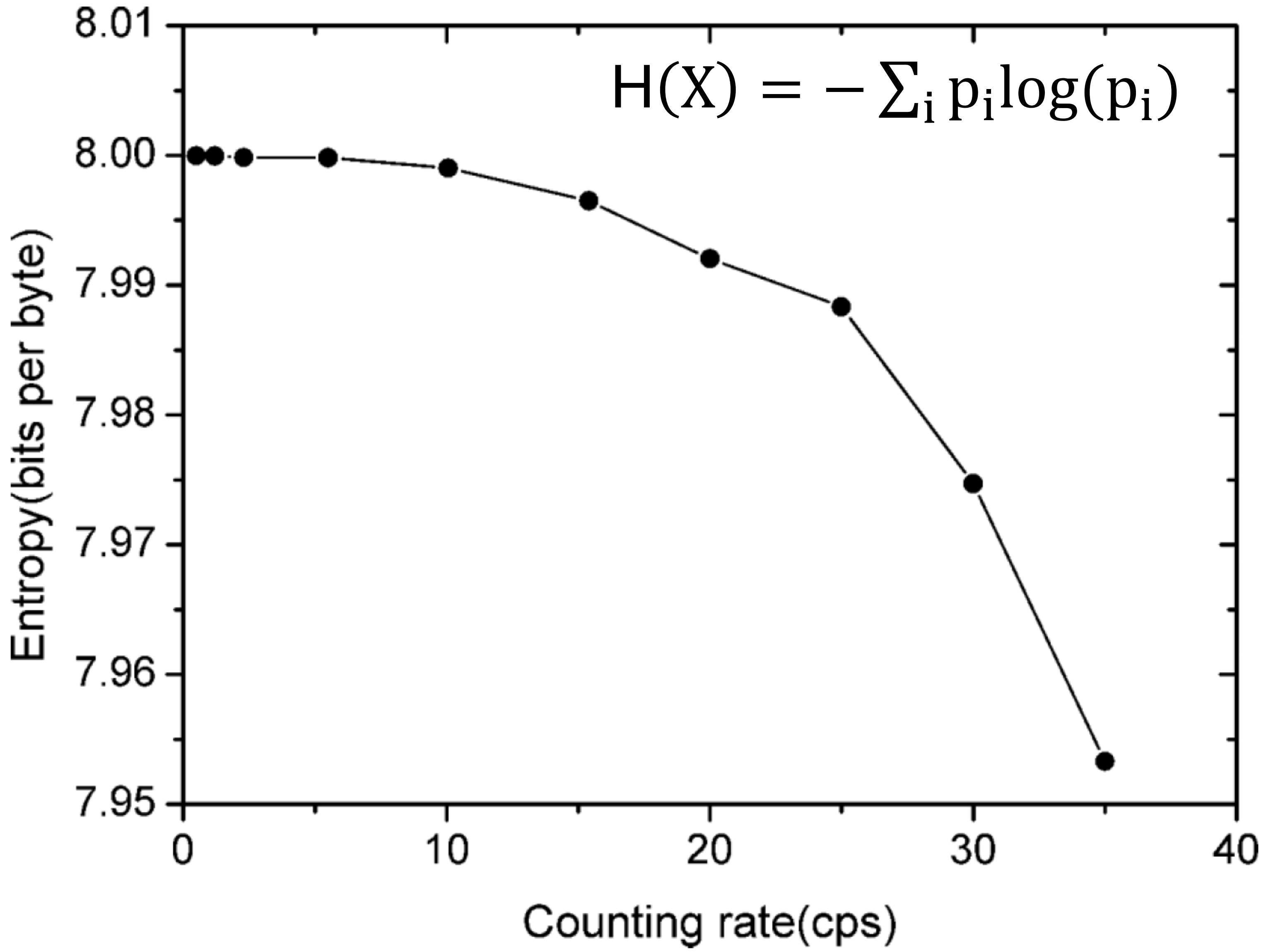
**Very low event rate**
**$\lambda T_w \ll 1$**

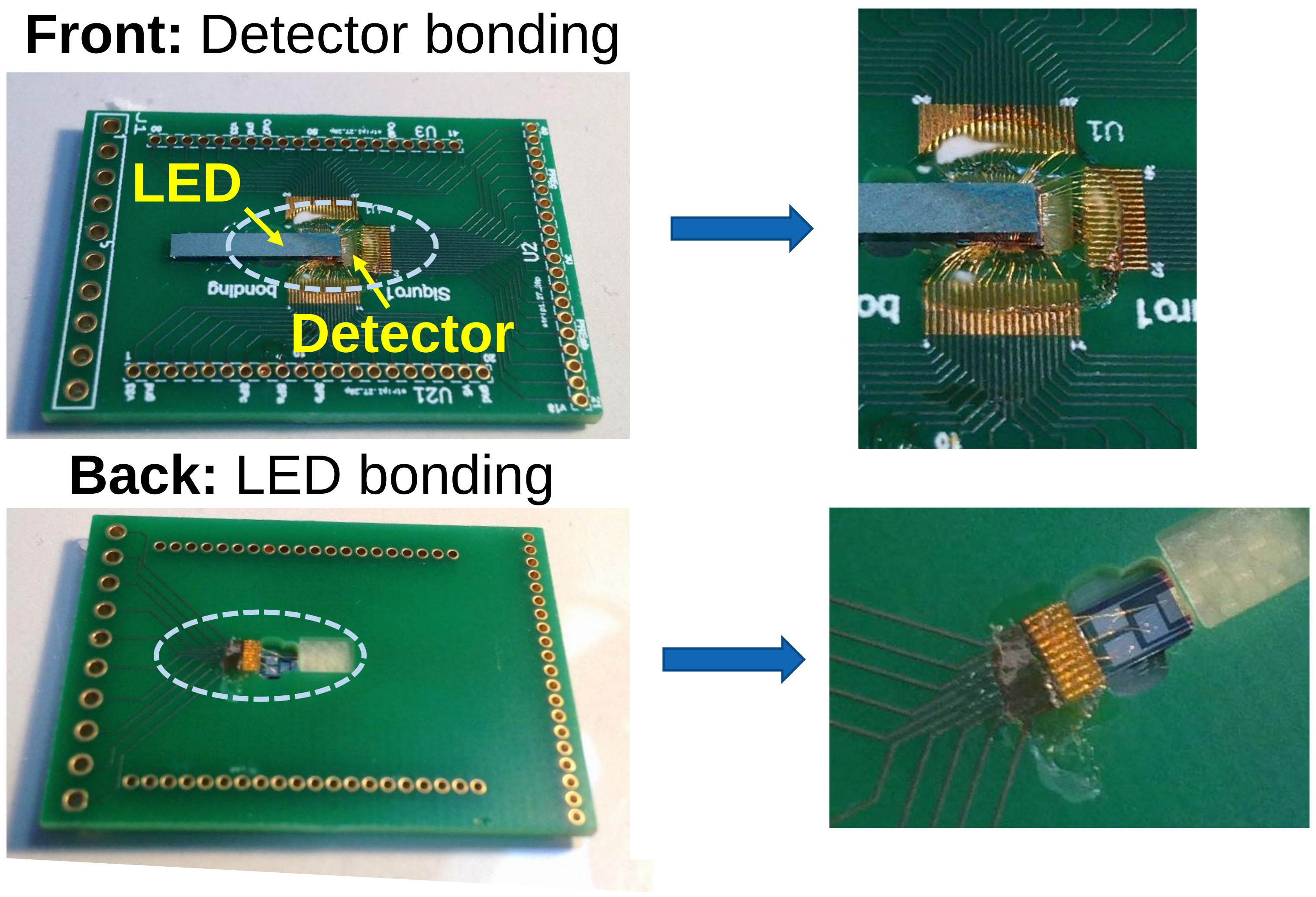
$$P\{t \le \tau |\, N(T_w) = 1\} = \frac{P\{t \le \tau, N(T_w) = 1\}}{P\{N(T_w) = 1\}} = \frac{P\{N(\tau) = 1, N(T_w) - N(\tau) = 0\}}{P\{N(T_w) = 1\}} =$$

$$= \frac{\lambda\tau \cdot e^{-\lambda\tau} \cdot e^{-\lambda(T_w - \tau)}}{\lambda T_w \cdot e^{-\lambda T_w}} = \frac{\tau}{T_w}$$

→If the flux is so low to ensure up to 1 detection per $T_w$, the arrival time is uniformly distributed in $T_w$

# QRNG based on arrival time
## Multi-bit generation



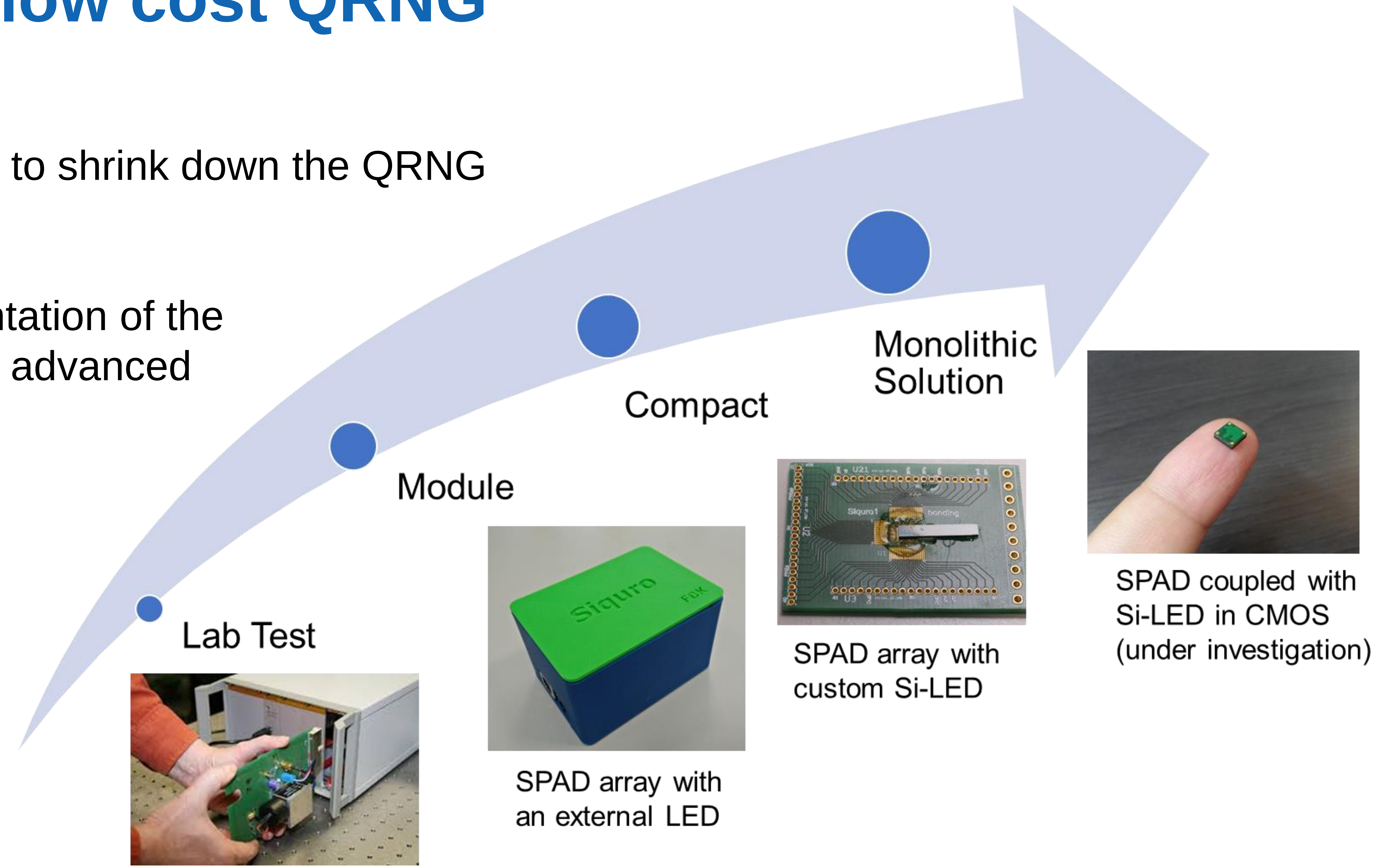

Use of an external Si-nc LED to design a compact design

*[Mas19]*

# Monolithic QRNG
## Towards a low cost QRNG

- Integration allows to shrink down the QRNG dimension and cost

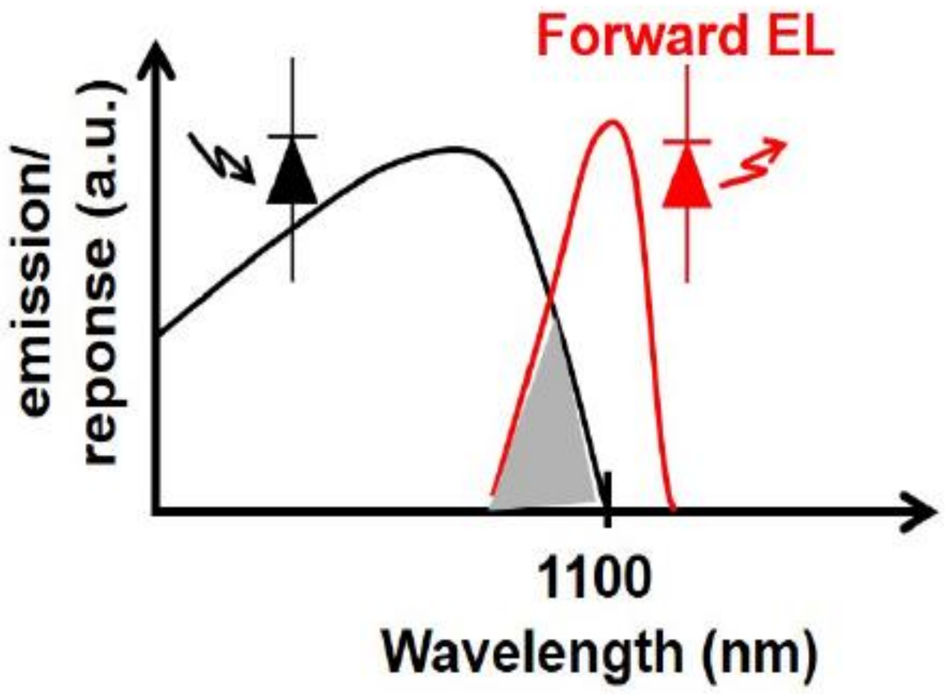
- **Target:** implementation of the QRNG in a standard advanced technology node

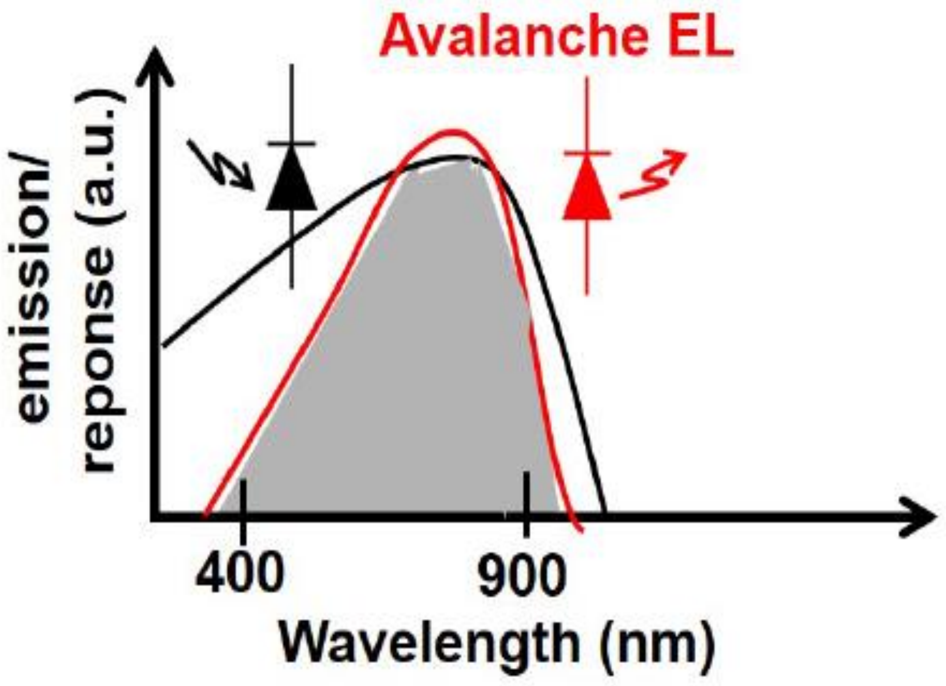


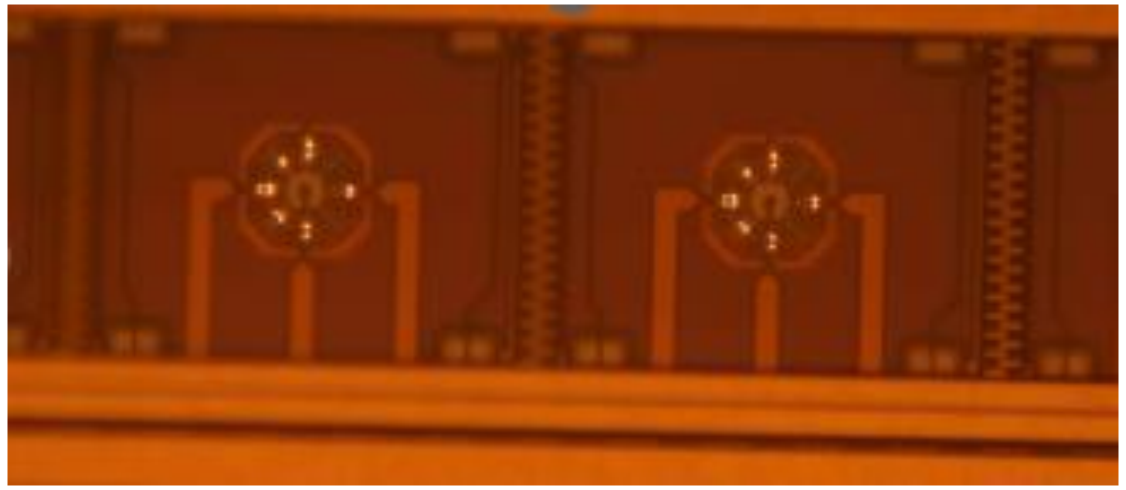
# Monolithic QRNG
## In-silicon source of light

Implementation of a silicon LED:

- Forward emission: peaked at NIRevice description → poor matching with the detector
- Reverse-avalanche emission: broad range with better matching with detector



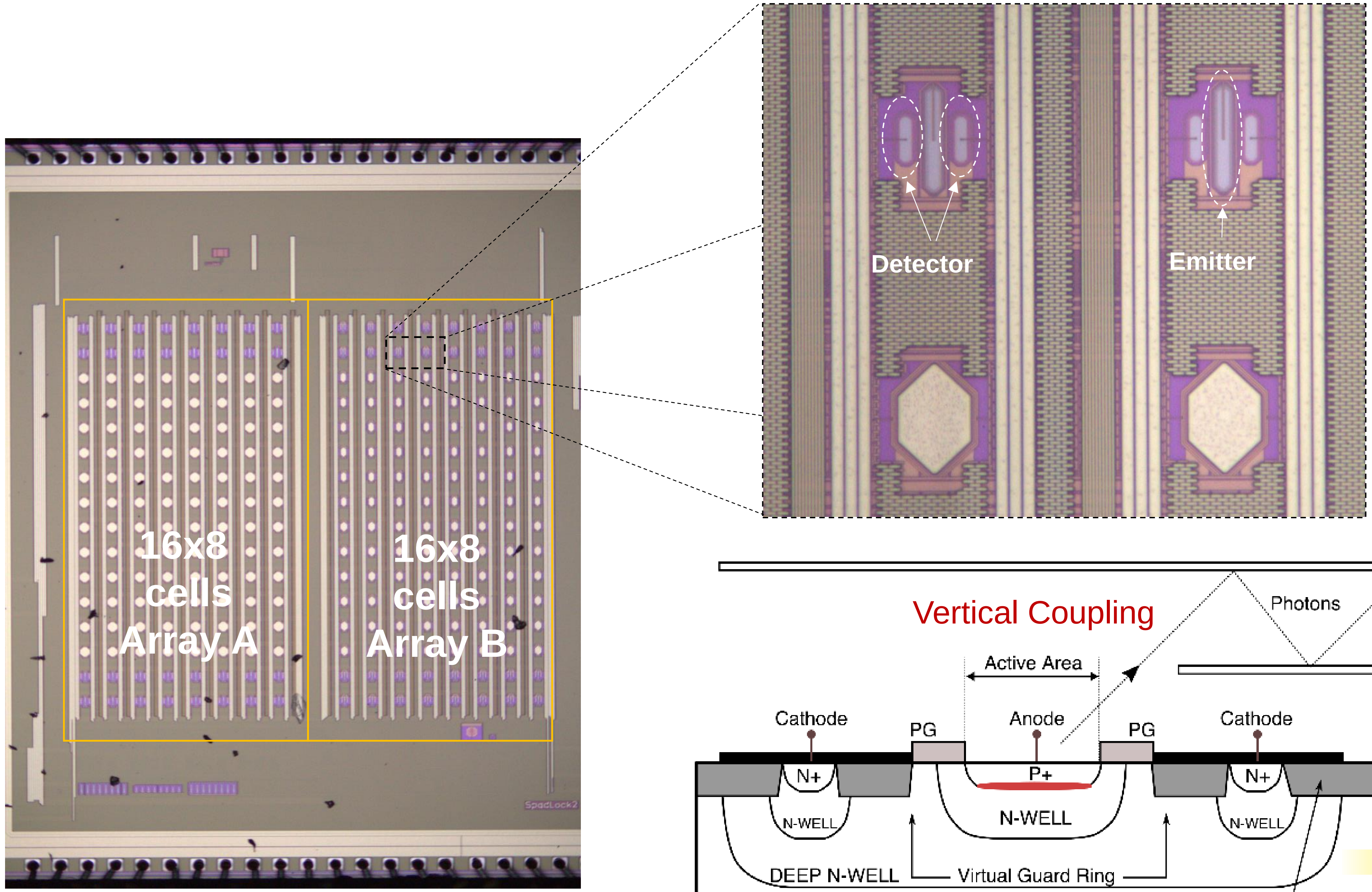



Visible light observed in test structures and capture through a microscope

*[Khan15] [Ace20]*

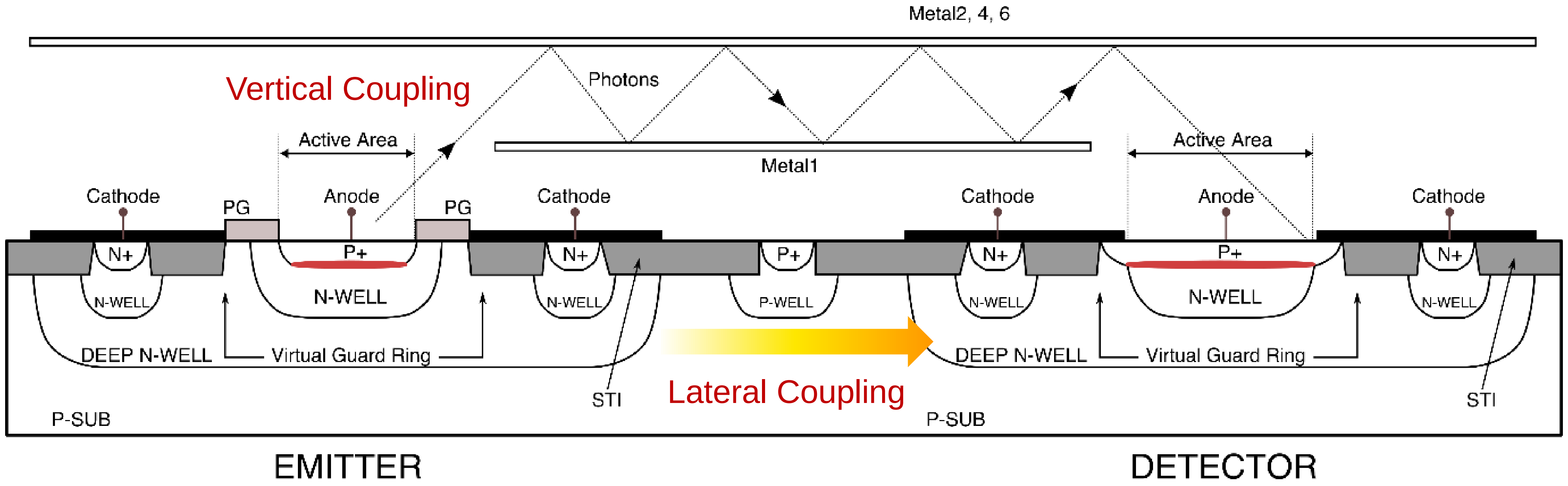
# Monolithic QRNG
## In-silicon source of light



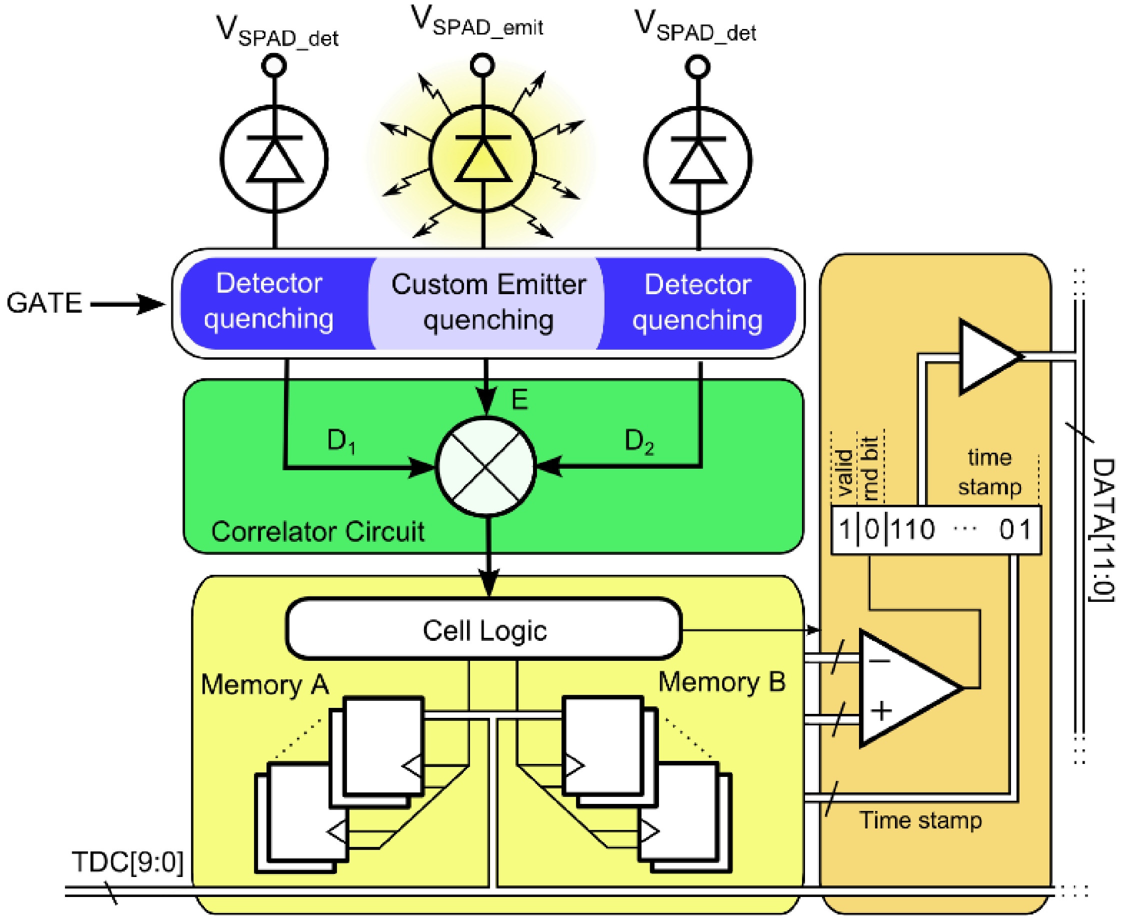

Two arrays of 18x8 cells, each having a couple of SPADs as detector and a central SPAD as an emitter of light

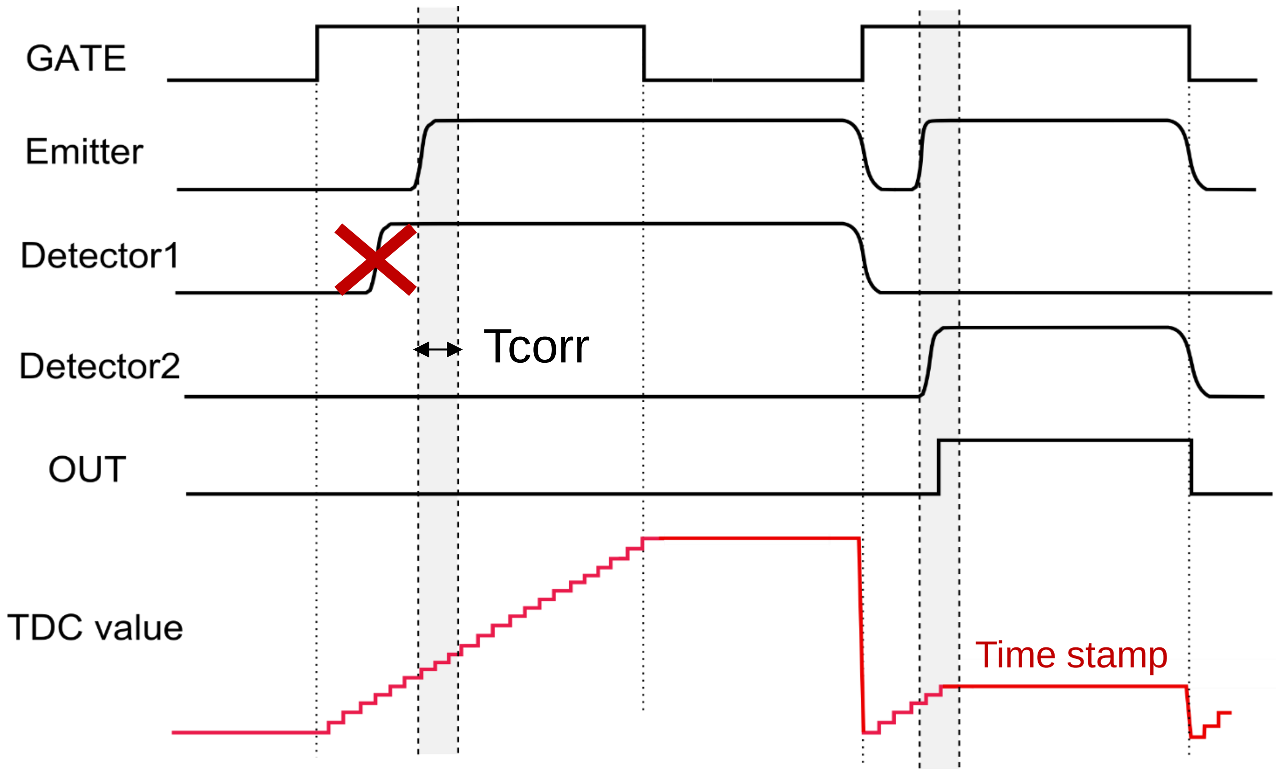
The emission of light is controlled by means of electrical parameters

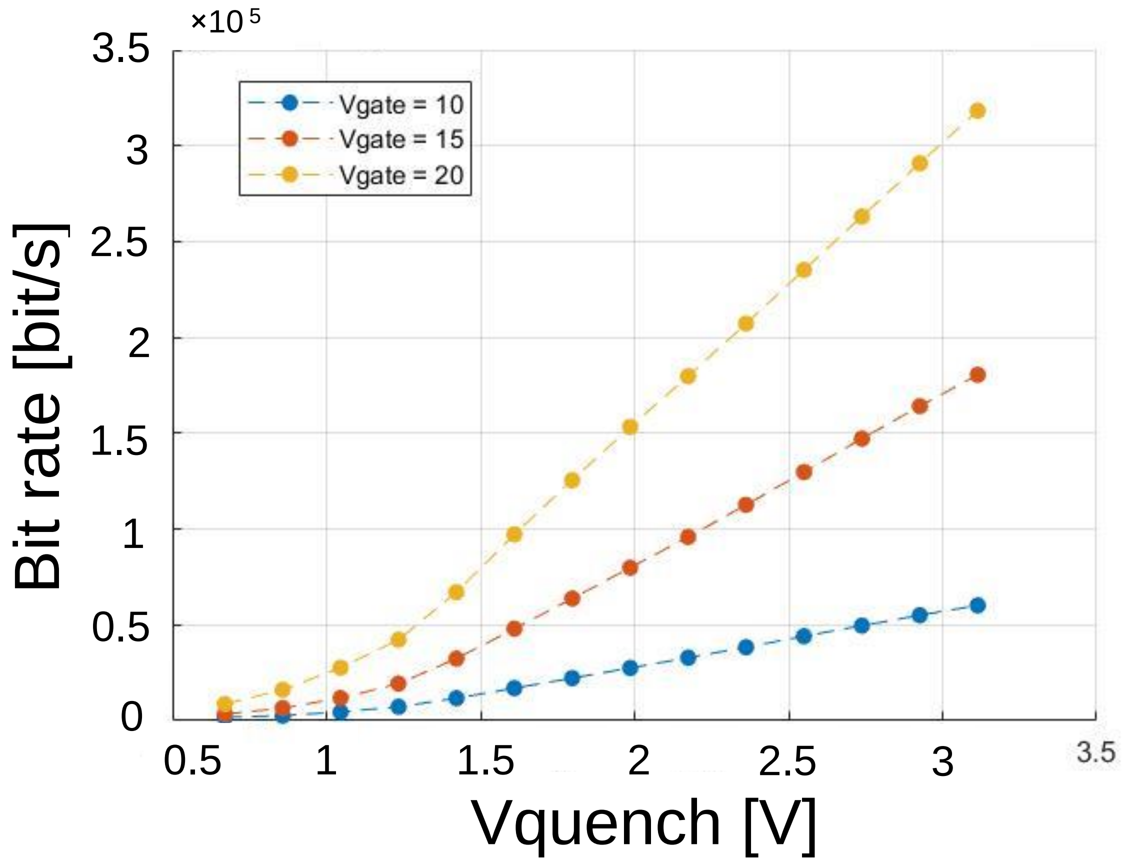
Optical cross-coupling is enhanced by using a top metal shield



# Monolithic QRNG
## In-silicon source of light




Every cell has proper circuit to control the light emission (custom emitter quenching) and a correlator circuit to exclude dark events for the generation of random numbers.

Achieved speed is ~ 400 Kbps

# QRNG review
## Conclusions

- Optical QRNG based on SPAD have shown encouraging results
- Different approaches has been shown with pros and cons
- SPAD-based QRNG can be potentially integrated in a standard state of the art CMOS technology
- SPAD-based QRNG can be implemented in an array to speed-up the generation (up to 5 Gbps have been demonstrated)
- A bit of effort has to be spent in order to increase the actual TRL of this technology

# Thank you

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