On Jitter Transfer in Ring Oscillators and Comprehensive Modelling of 1/f Noises

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[1]The work done during research stay at Hubert Curien Laboratory

Outline

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About Differential Measurement

- Measures the effect by *comparing sensing and referencing signals*
- Widely used, from explosives detection to random number generation

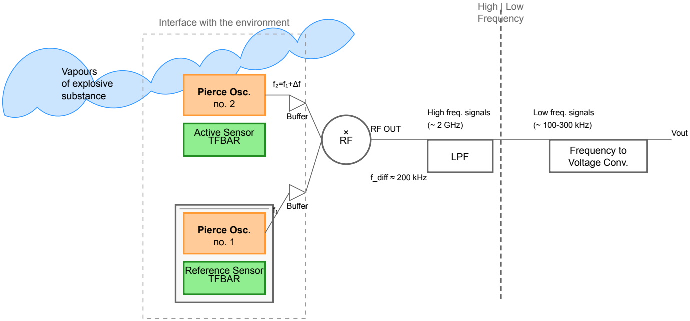


Figure: The mechanism of explosives detection [Vasile et al., 2021]. The sensor's resonating frequency is changed by the mass of attached molecules of explosives. The change can be detected by comparison with a not exposed reference sensor.

Differential Measurement in Oscillatory TRNGs

- Bits are generated by sampling one signal with another, both noisy (!)
- Two noisy signals approximated by a noisy-free + double-noise setup
- *Approximation enables analyses* of TRNGs [Baudet et al., 2011]

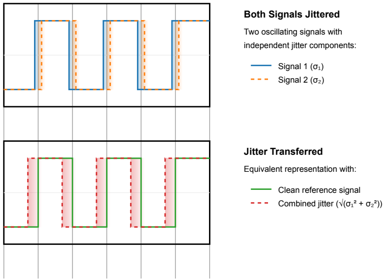


Figure: Jitter Transfer in Ring Oscillators. Two noisy signals are approximated by a noisy-free and double-noisy one, enabling security analysis.

The Challenge

- Key question: is this correct?
- Why it matters: critical for quantitative security evaluation
- Mathematical formulation: two noisy oscillatory signals

$$
\begin{aligned}
s_0(t) &= w(\phi_0 + f_0 t + \xi_t^0) \\
s_1(t) &= w(\phi_1 + f_1 t + \xi_t^1),
\end{aligned}
$$

  s_1 sampled at the edges of s_0. Here f_i are frequencies, ϕ_i are initial locations, and ξ_t^i are Brownian motions with volatility σ_i modelling phase modulation.
- How do they combine in sampling ?

Key Result: Model of Jitter Transfer

Under the assumption $\sigma_0^2 \ll f_0$:

- Sampling bits from two noisy oscillators equivalent to sampling from:
  - Clock s_0 being jitter-free
  - Signal s_1 having volatility:

$$\sigma = \sqrt{\frac{f_1^2}{f_0^2}\sigma_0^2 + \sigma_1^2}$$

- Error bounds available through normal approximation quality
- Critical for analysing multi-oscillator TRNGs

Difficulty

Analysing arrival times of the rising edges is hard (hitting times).

Exact Statistical Properties

Key Distributions:

- Phase distribution at sampling time T_k:

$$\Phi_1(T_k) \sim \mathrm{N}(\phi_1 + f_1 T_k, \sigma_1^2 T_k)$$

- Clock edge timing for the k-th rising edge:

$$T_k \sim \mathrm{IG}\left(\frac{k-\phi_0}{f_0}, \frac{(k-\phi_0)^2}{\sigma_0^2}\right)$$

- Period distribution between edges:

$$T_{k+1} - T_k \sim \mathrm{IG}\left(\frac{1}{f_0}, \frac{1}{\sigma_0^2}\right)$$

Important Note

These exact formulas enable precise security analysis

Normal Approximation

- When $\frac{\sigma_0^2}{f_0} \to 0$, we have:

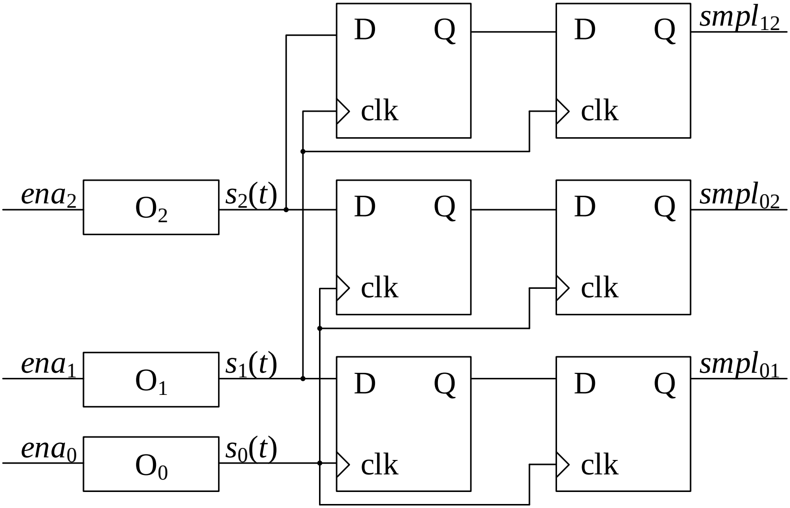
$$\frac{\phi_1(T_{k+1}) - \phi_1(T_k) - \mu}{v} \xrightarrow{d} \mathrm{N}(0,1),$$

  where:
  - $\mu = \frac{f_1}{f_0}$
  - $v = \sqrt{\frac{\sigma_1^2}{f_0} + \frac{f_1^2\sigma_0^2}{f_0^3}}$
- Convergence is uniform in σ_1, f_1
- Quality improves as jitter-to-period ratio decreases

Applications

- Novel tool for analysing multi-ring oscillator TRNGs
- Enables:
  - Quantitative differential jitter measurements
  - Individual oscillator volatility recovery
  - More accurate entropy rate computation
- Two practical methods:
  - Method 1: Assumes linear jitter variance with period
  - Method 2: No assumptions, requires extra hardware

Implementation Details



Implementation Results

Key Findings:

- Hardware tests on Intel Cyclone V FPGA
- Method 1 assumptions not always valid:
  - Significant discrepancies observed
  - $\sigma_0(T_0)$ can vary by factor of 2
- Method 2 more reliable:
  - Consistent results across experiments
  - Proven numerically stable
  - Small hardware overhead (one extra flip-flop)

Details and Future Work

- For technical details, please refer to [Lubicz and Skorski, 2024]
- The techniques use Laplace Transform and results on hitting times
- Extensions to generic Gaussian Processes via [Decreusefond and Nualart, 2008]...?

Outline

Challenge

- Randomness used in games, simulations, cryptography...
- Entropy models needed for security (NIST-90B, AIS20/31)
- Tests confirm pseudorandomness, many fast proposals lack guarantees!
- What is a good stochastic model for voltage/quantum RNGs?

Oscillatory TRNG Basics

- Periodic signal with phase noise:

$$y(t) = \sin(2\pi f t + \xi(t))$$

- Bit extraction by subsampling and checking low/high state:

$$b_n = \begin{cases} 1 & y(nT) > 0 \\ 0 & y(nT) \leq 0 \end{cases}$$

- Security depends on phase noise $\xi(t)$ modelling
- Similar modelling possible for electric field (quantum effects)

Five-Power Noise Law

Per empirical evidence [Howe et al., 1981], for hardware-dependent constants h_α:

- Instantaneous frequency spectrum:

$$S_{\dot{\xi}}(\omega) \approx \sum_{\alpha=-2}^{2} h_\alpha \omega^\alpha$$

- Phase spectrum:

$$S_{\xi}(\omega) \approx \sum_{\alpha=-2}^{2} h_\alpha \omega^{\alpha-2}$$

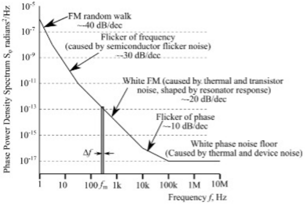


Figure: Five-Power Spectral Law (www.harmanluxuryaudionews.com)

Novel Gaussian Process Phase Model

Assumption: Phase follows Fractional Brownian Motion [Lévy, 1953]

$$\xi(t) = \frac{1}{\Gamma(H+1/2)} \int_0^t (t-u)^{H-1/2} \mathrm{d}B_u$$

where B_u is Brownian motion and H is the Hurst–Hölder exponent.

Key Properties:

- Extends Barnes and Allan's proposal [Barnes and Allan, 1966]
- Non-stationary Gaussian Process
- Flexible to match expected spectral law
- Posterior is Gaussian with uncertainty estimates

Covariance Properties

- The covariance equals

$$\mathbf{Cov}[L_H(t), L_H(t+\tau)] = \frac{2t^{H+\frac{1}{2}}(t+\tau)^{H-\frac{1}{2}}\,{}_2F_1\left(1, \frac{1}{2}-H; H+\frac{3}{2}; \frac{t}{t+\tau}\right)}{\Gamma(H+1/2)^2(2H+1)}.$$

- Important special cases
  - For $H=1$ (flicker noise)

$$\mathbf{Cov}[L_0(t), L_0(t+\tau)] = \frac{1}{\pi}\left(\sqrt{t}\sqrt{t+\tau}(2t+\tau) - \tau^2 \tanh^{-1}\left(\frac{\sqrt{t}}{\sqrt{t+\tau}}\right)\right),$$

  - For $H=1/2$ (white noise)

$$\mathbf{Cov}[L_1(t), L_1(t+\tau)] = t.$$

- Sampling can be easily implemented, also on GPU!

Path Samples

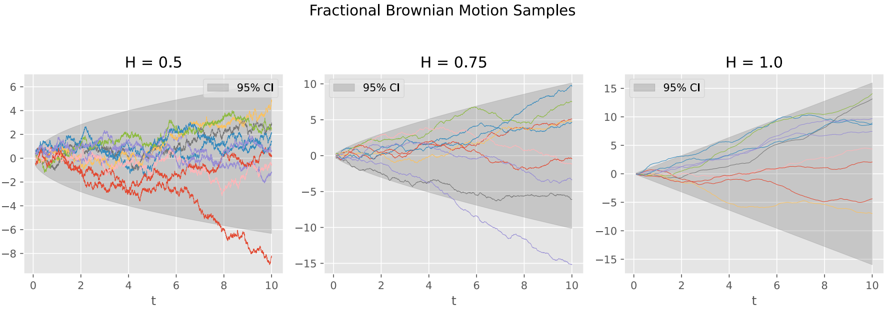


Figure: Path Samples using Cholesky's Decomposition.

Spectral Properties

Flexibility of Gaussian Process matches empirical law:

- Wigner-Ville spectral density (time-averaged):

$$S_{\xi}^{WV}(\omega) \approx \omega^{-2H-1}$$

- $H = 1$: flicker frequency modulation ($\alpha = -1$)
- $H = 1/2$: white noise frequency modulation ($\alpha = 0$)

Power Spectral Density

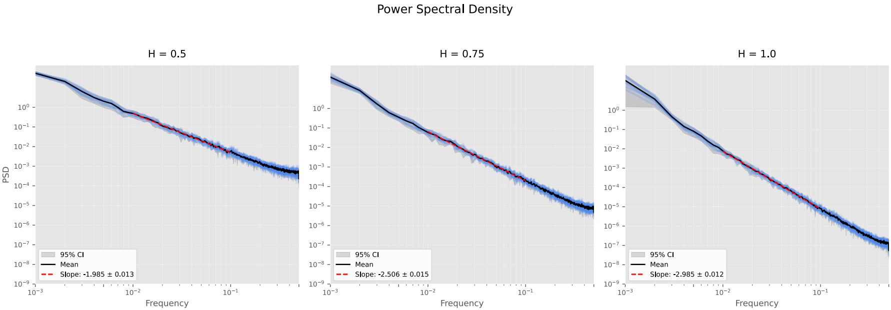


Figure: Spectral Density using Welch's Estimator.

Security Analysis

Approach:

- Focus on flicker and white noise components
- Evaluate unpredictability of next phase $X = X_n$
- Consider attacker knowing past locations $Y = X_1, \ldots, X_{n-1}$
- Use Schur-complement leakage rule:

$$\mathbf{Cov}[X] = \mathbf{Cov}[X] - \mathbf{Cov}[Y, X]^{\top}\mathbf{Cov}[Y]^{-1}\mathbf{Cov}[Y, X]$$

Key Findings:

- Leftover variance stabilizes away from zero
- Implies unpredictability and non-trivial security
- Strengthens the Monte-Carlo approach [Peetermans and Verbauwhede, 2024]

Leakage Resiliency

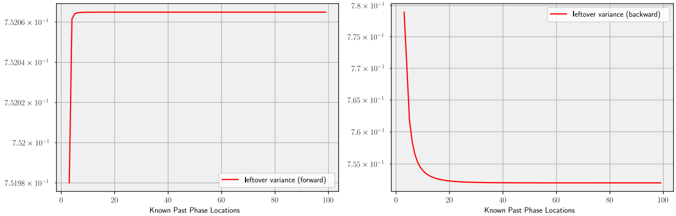


Figure: Leftover variance (conditioned on past locations).

Summary

- Precise model for jitter transfer, application to multi-ring TRNG
- Comprehensive noise modelling using fBm, security through leftover variance

Details and Future Work

- For technical results, see [Skorski, 2024]
- Implementation PoC with GPU acceleration
  https://www.kaggle.com/code/mskorski/fractional-brownian-motion?scriptVersionId=207845405
- TBD: Accurate determination of hardware constants
- TBD: Formal proof of bounded leakage conjecture for leftover variance
- TBD: Optimization of sampling efficiency

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Outline

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Thank you!

Questions?